Deriving Abstract Interpreters

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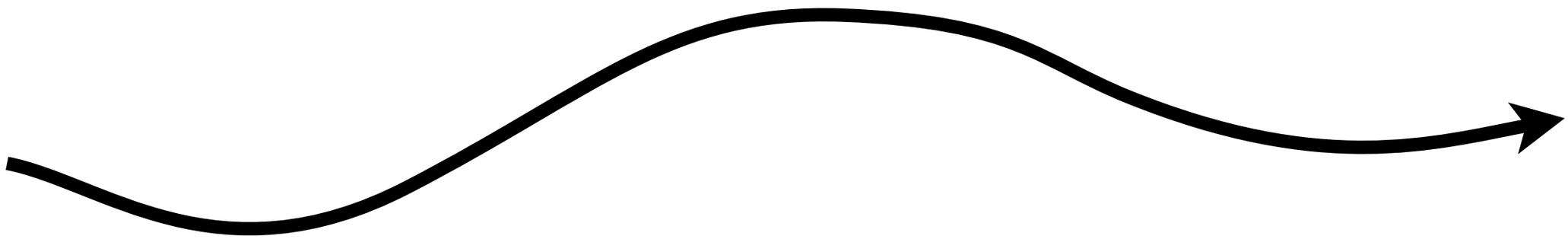
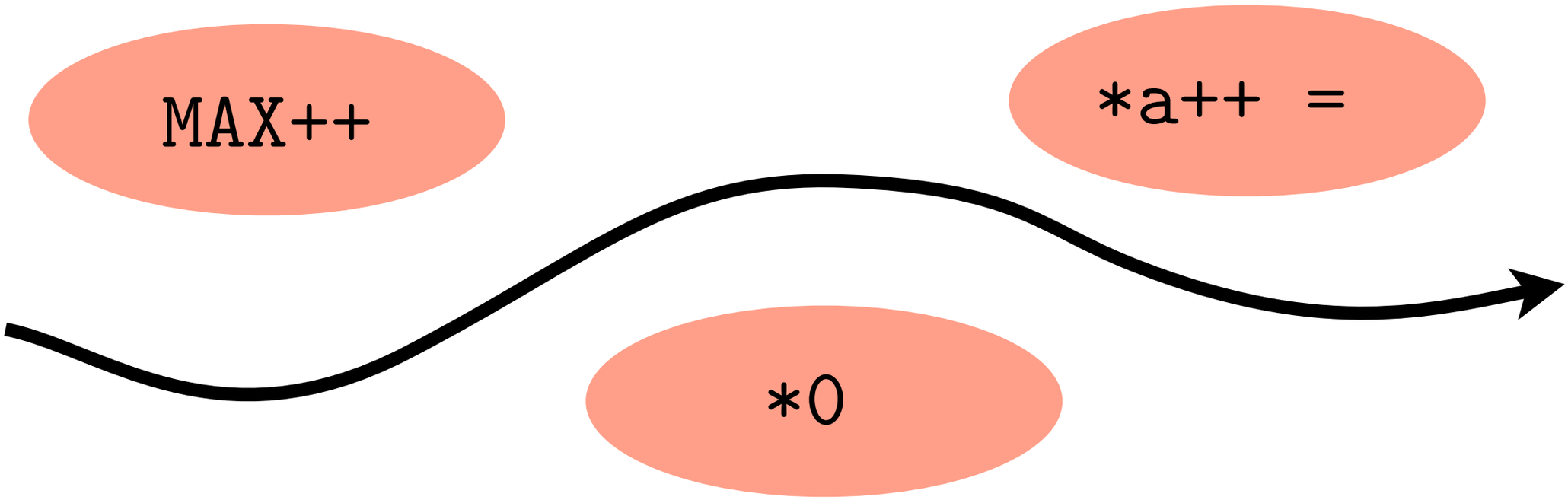
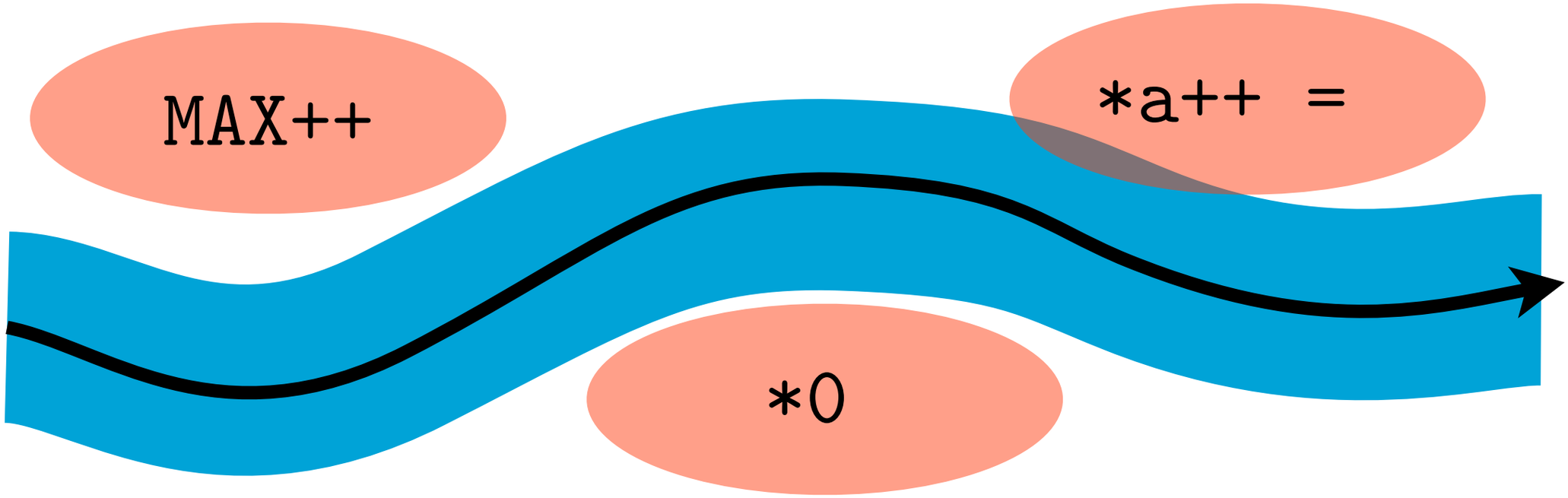
static analysis




MAX++
*a++ =
*0


MAX++
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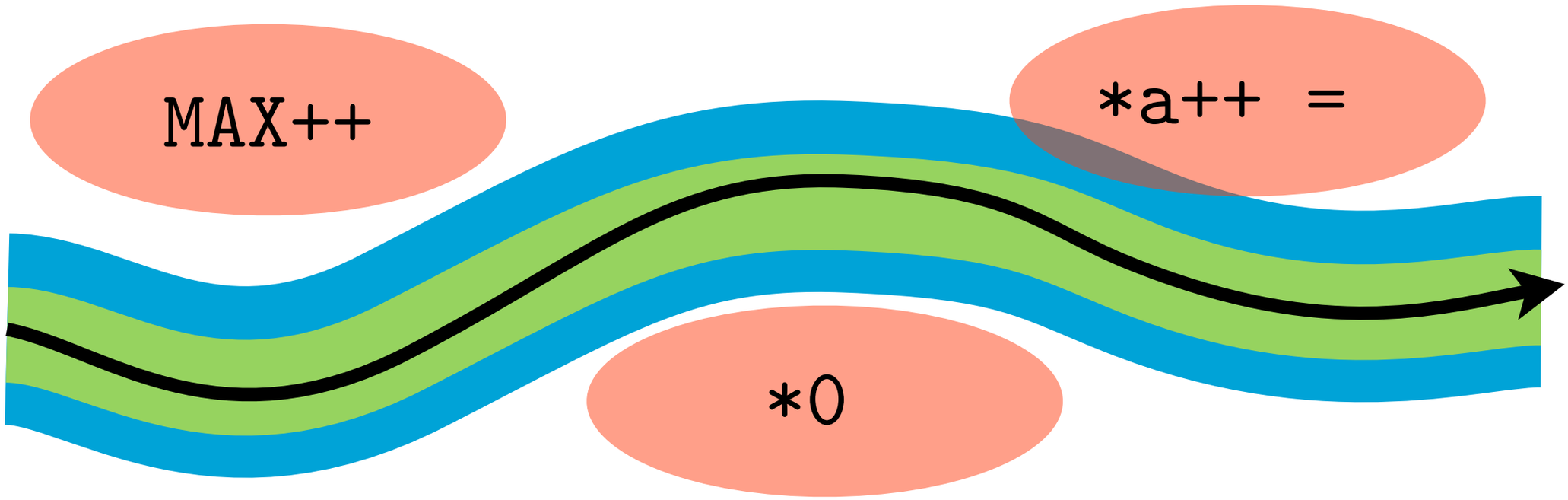
*0


MAX++
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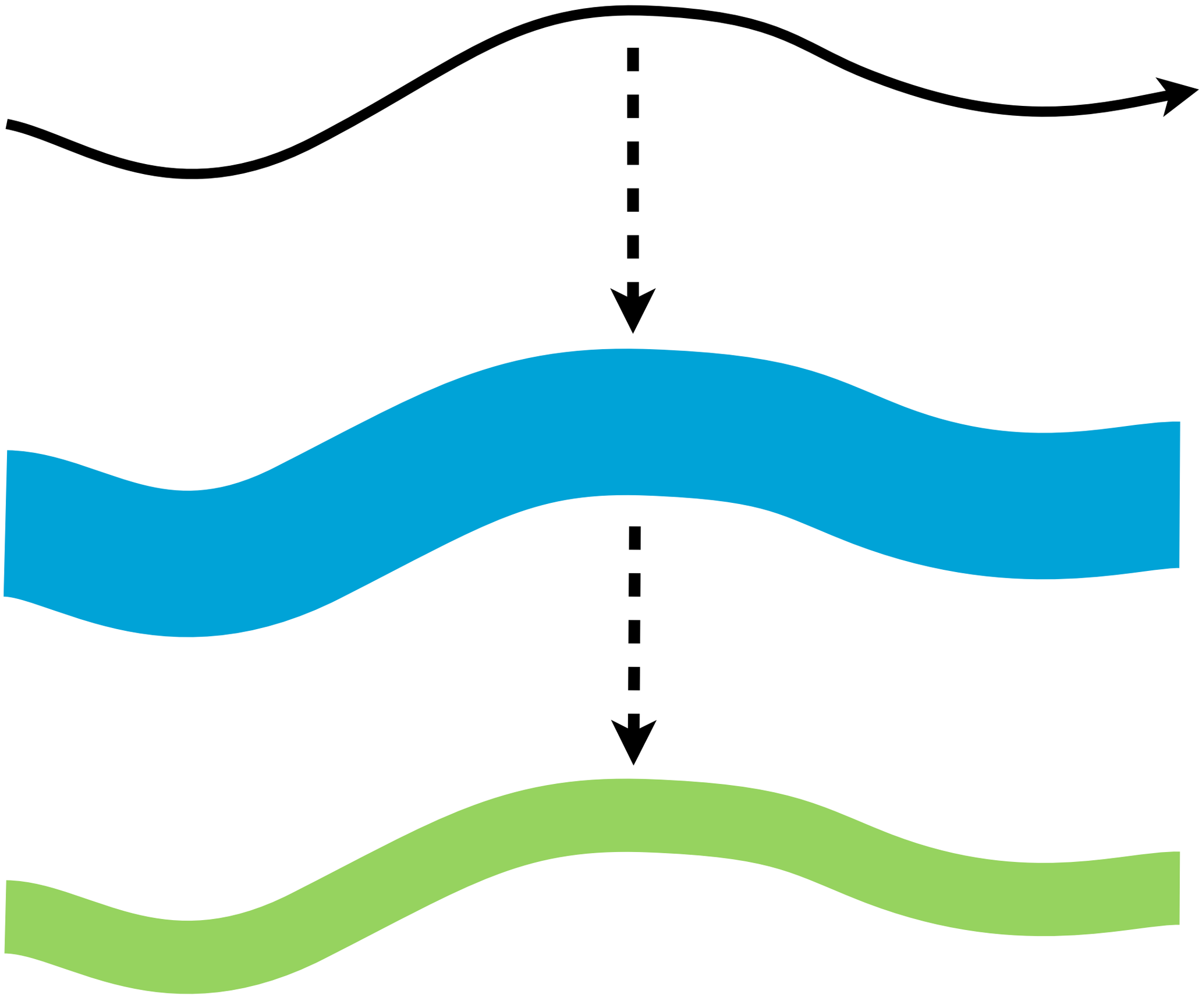


How do you teach this?

f

$\hat{f}$

g

$\hat{g}$

Years later...

h

h

$\hat{h}$

Madness

to this

Method

Method

to this

Madness

Outline

- Small steps
- CPS, 0CFA
- ANF, PDFA

Small-step analysis

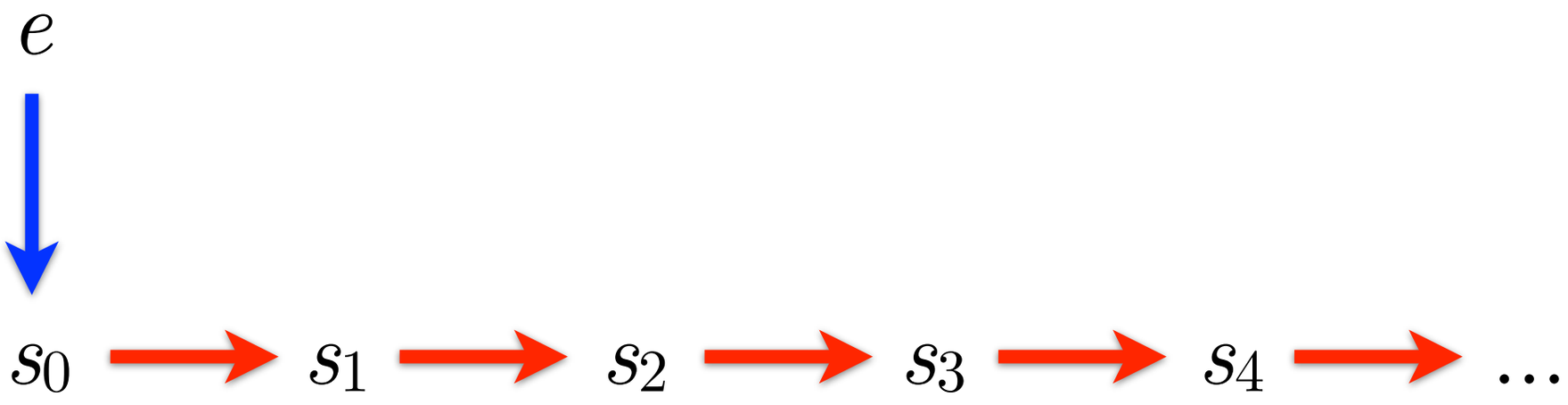
Concrete semantics

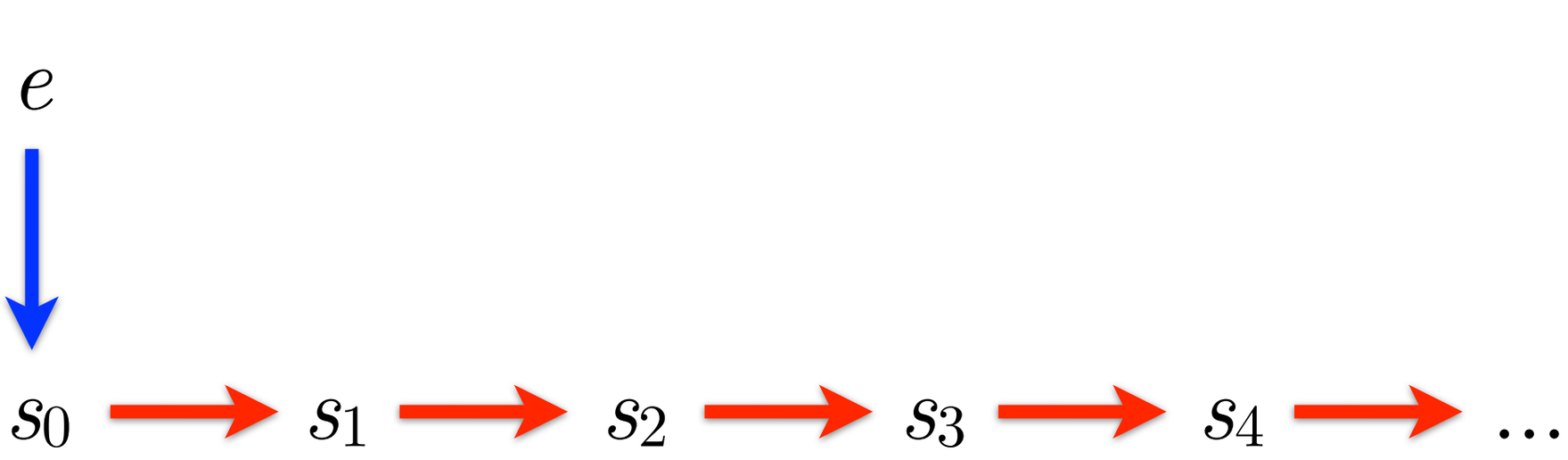
Concrete semantics

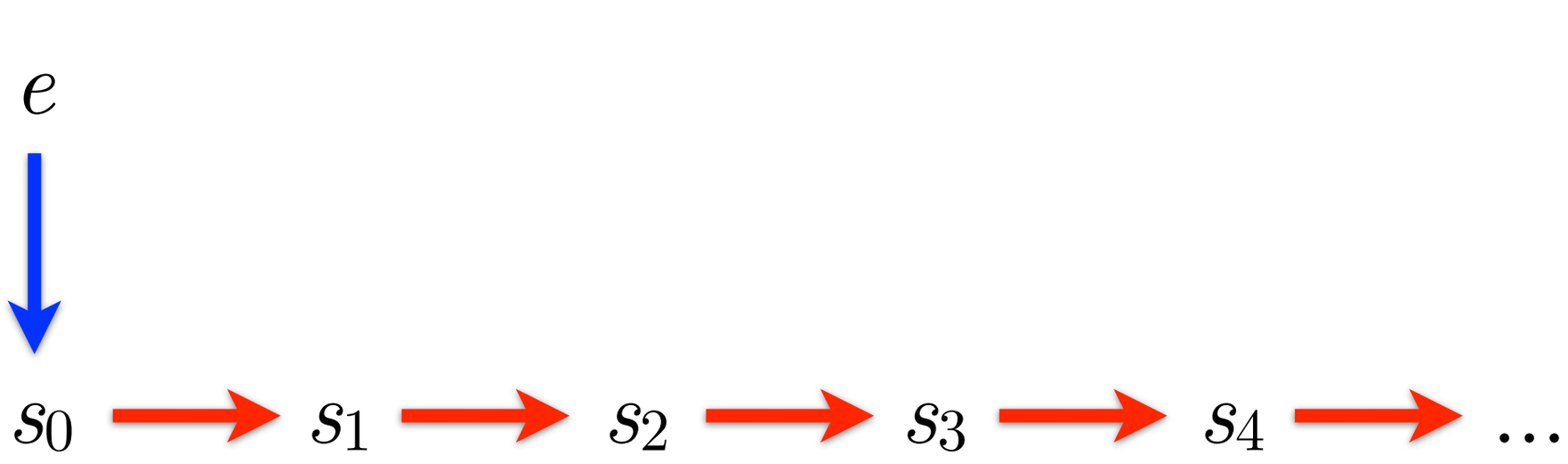
- Convert program e into machine state s_0

Concrete semantics

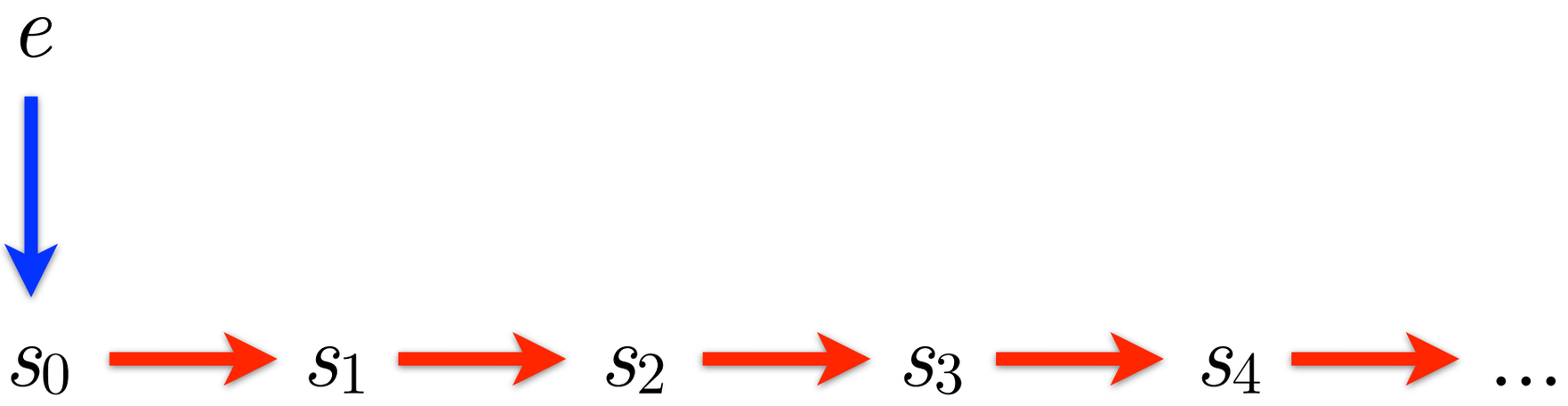
- Convert program e into machine state s_0
- Transition from state s_n to state s_{n+1}



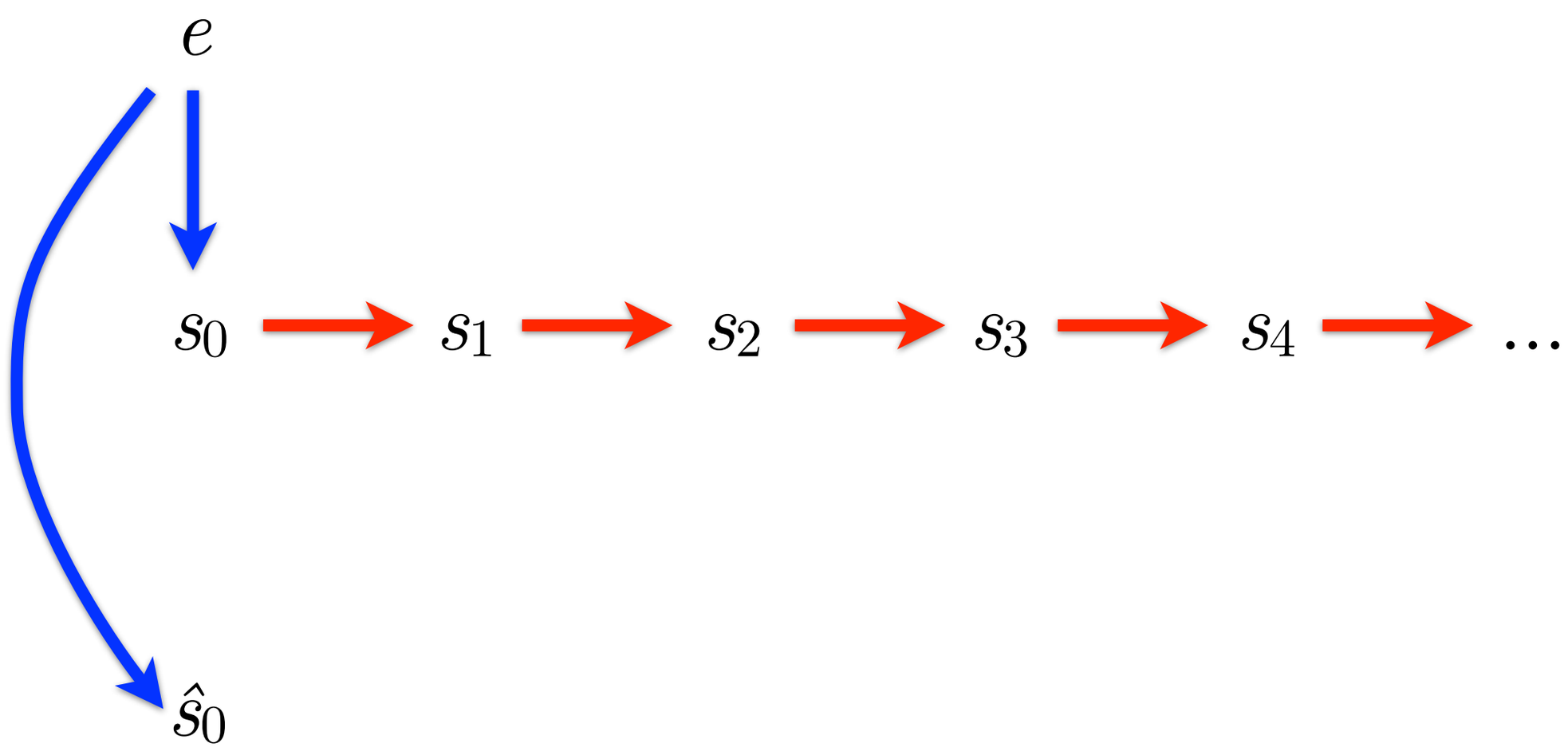

e
s_0
s_1
s_2
s_3
s_4
...


e
s_0
s_1
s_2
s_3
s_4
$\ldots$

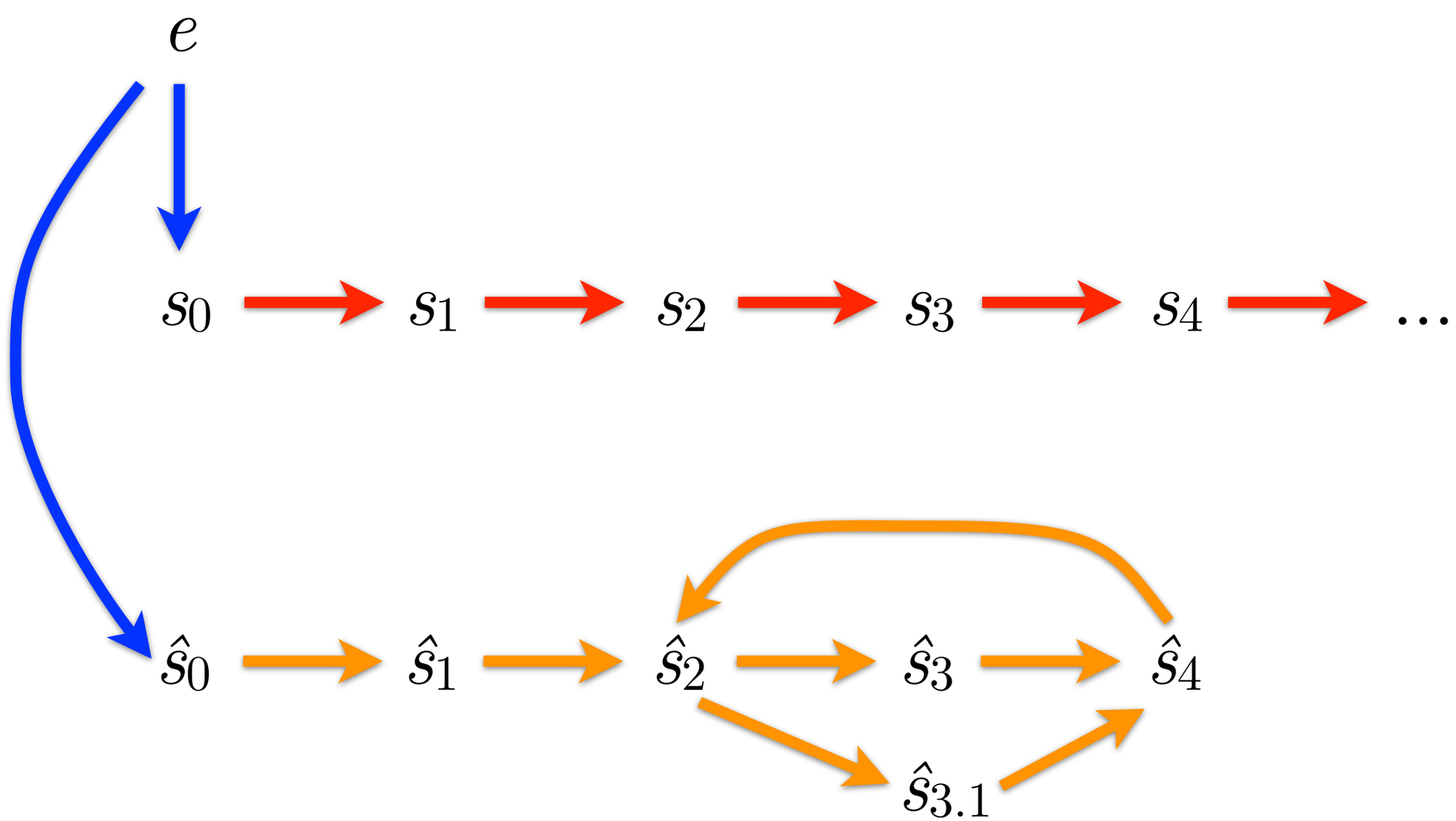
Abstract semantics



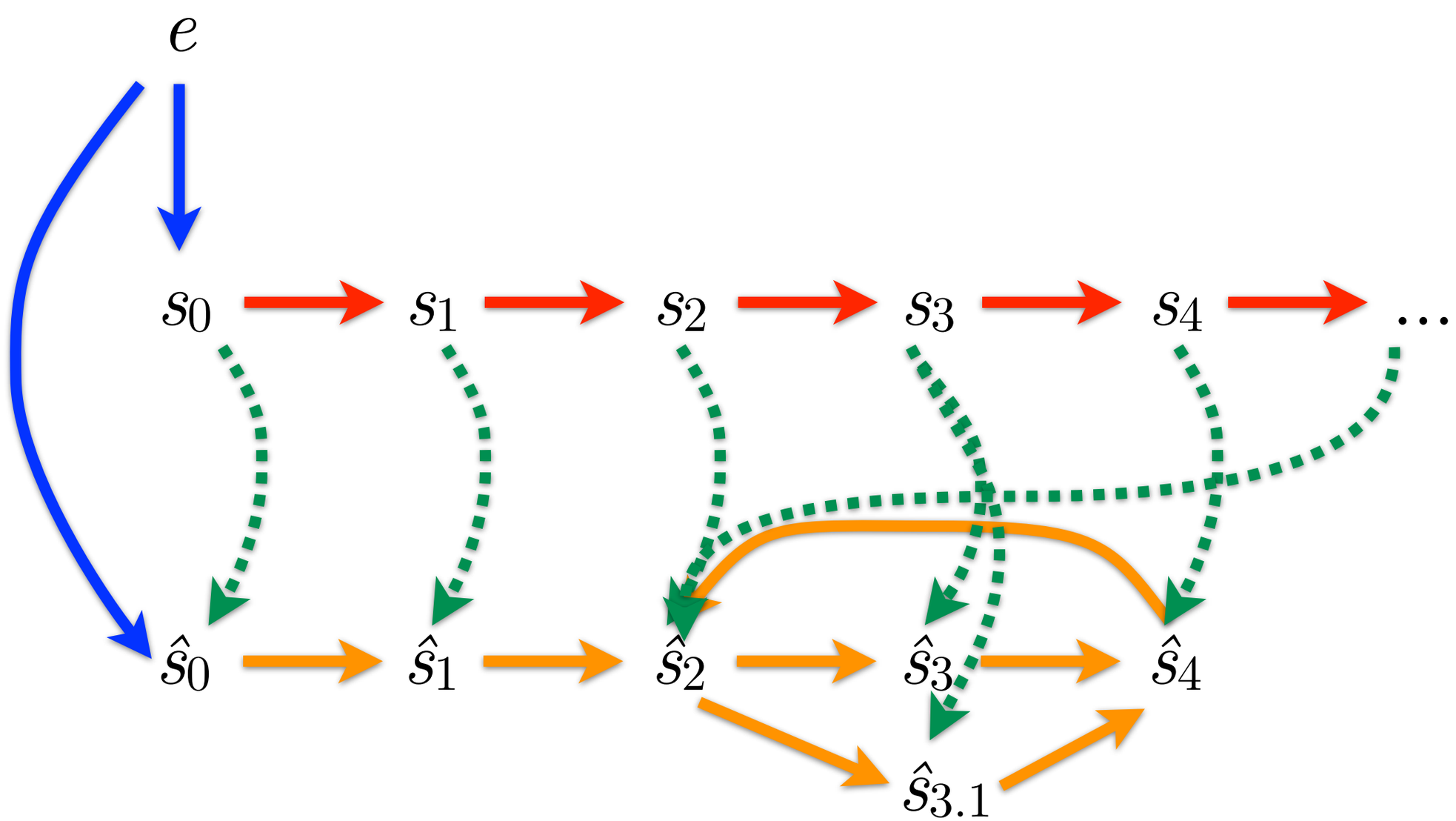
Abstract semantics



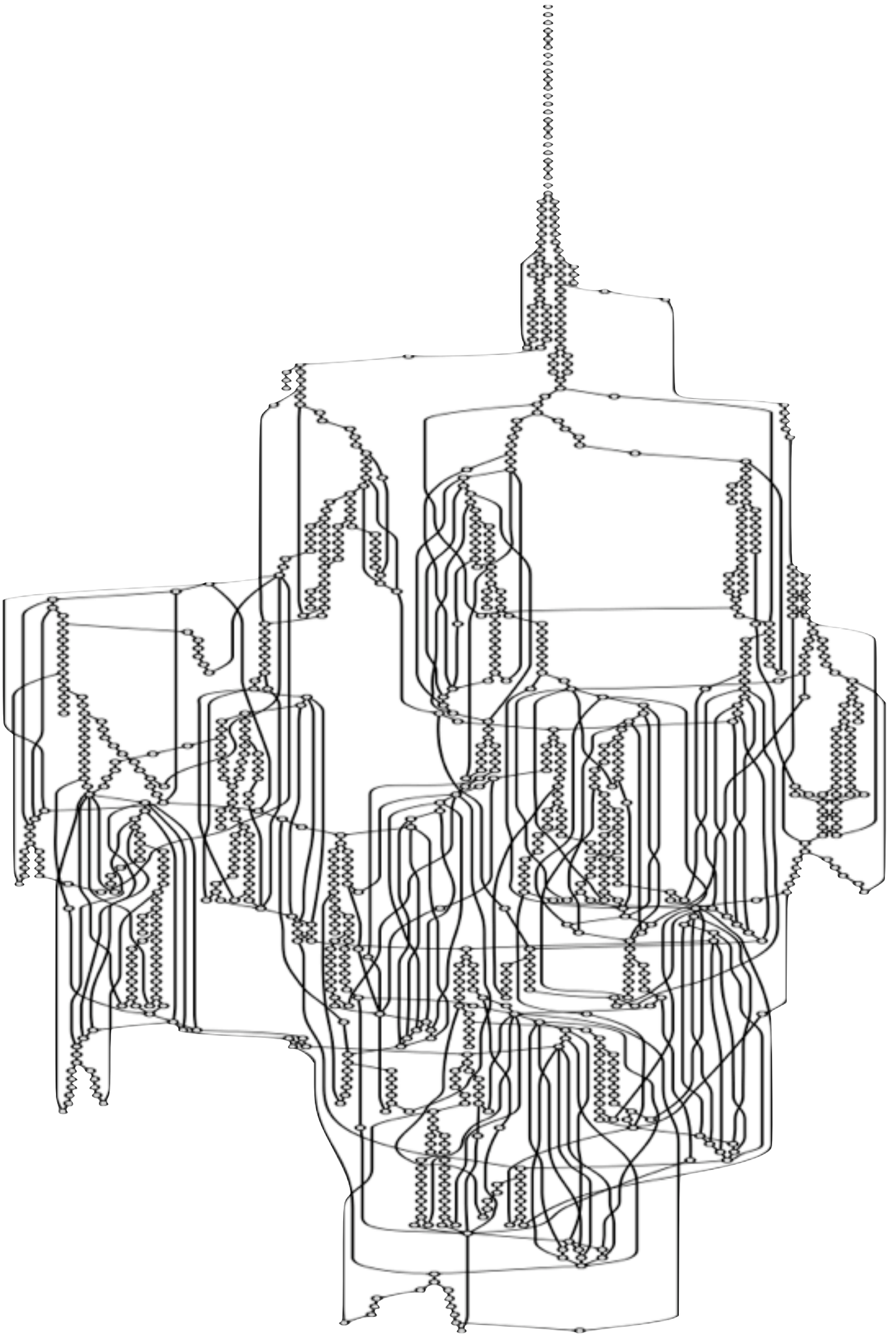
Abstract semantics



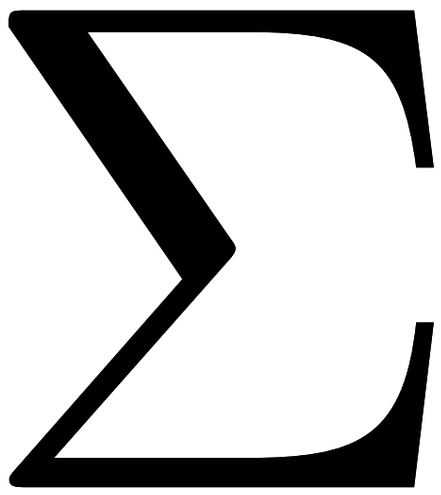
Abstract semantics



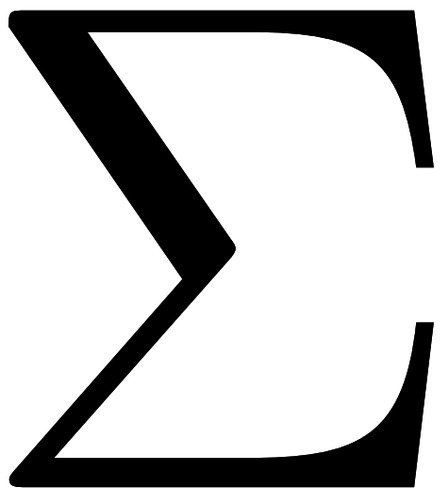
Theorem: The abstract simulates the concrete.



Components of small-step



$$(\Rightarrow) \subseteq \Sigma \times \Sigma$$



Σ

$$\alpha : \sum \to \hat{\sum}$$

$\hat{\Sigma}$

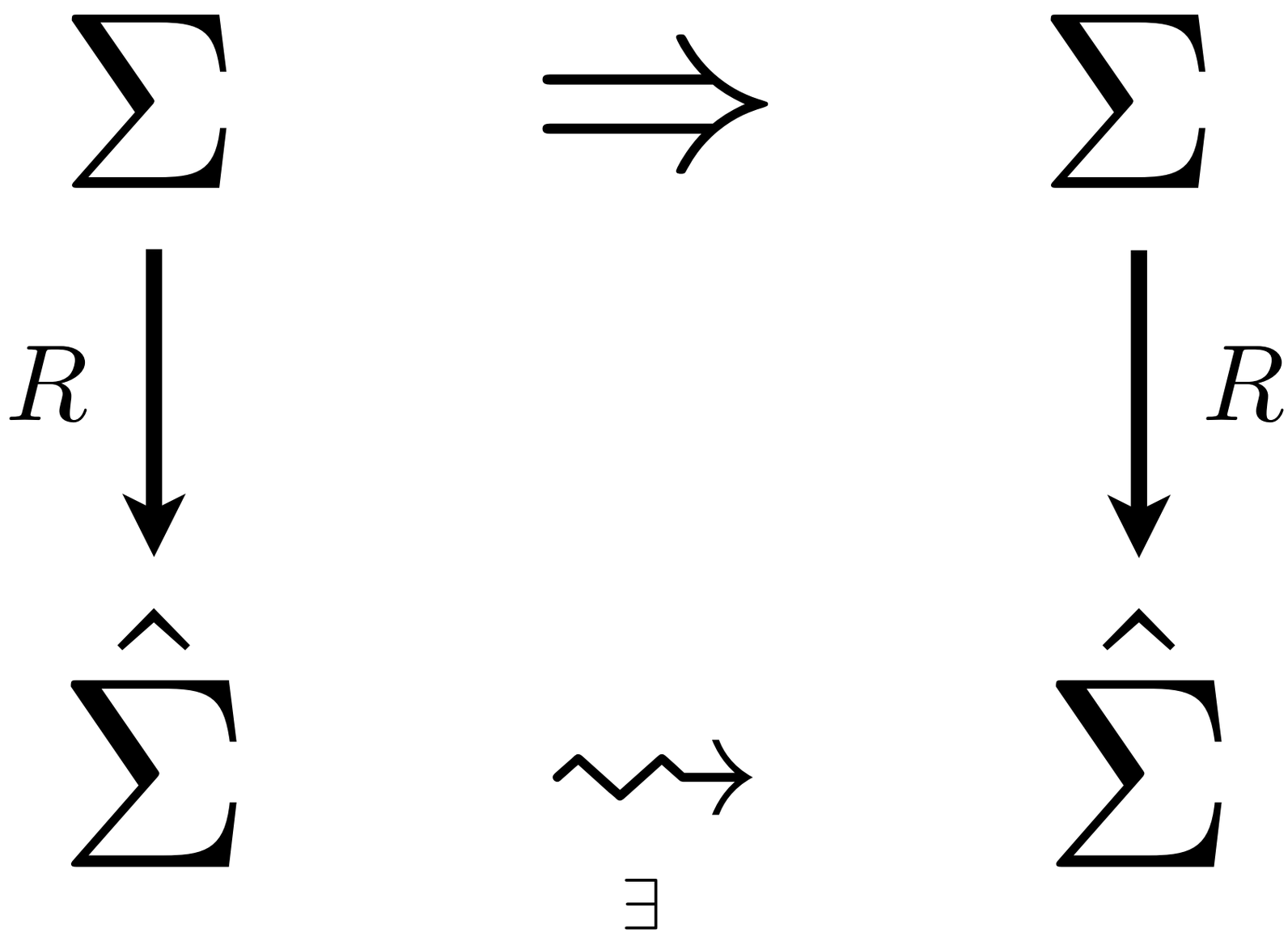
$\hat{\Sigma}$

$$(\rightsquigarrow) \subset \hat{\Sigma} \times \hat{\Sigma}$$

$$(\rightsquigarrow) \subseteq \hat{\Sigma} \times \hat{\Sigma}$$

$$(\Rightarrow) \subseteq \Sigma \times \Sigma$$

$$(\rightsquigarrow) \subseteq \hat{\Sigma} \times \hat{\Sigma}$$


Σ
$\Rightarrow$
Σ
R
R
$\hat{\Sigma}$
$\rightsquigarrow$
$\exists$
$\hat{\Sigma}$

$$R(\varsigma, \hat{\varsigma}) \text{ iff } \alpha(\varsigma) \sqsubseteq \hat{\varsigma}$$

$$(\sqsubseteq) \subseteq \hat{\Sigma} \times \hat{\Sigma}$$

$$\hat{L} = \mathcal{P}(\hat{\boldsymbol{\Sigma}})$$

Classical abstract interpretation?

$$\hat{L} = \mathcal{P}(\hat{\Sigma})$$

$$\hat{L} = \mathcal{P}(\hat{\boldsymbol{\Sigma}})$$

$$f : L \to L$$

$$\hat{f}(\hat{S}) = \{\hat{\varsigma}_0\} \cup \{\hat{\varsigma}' \mid \hat{\varsigma} \rightsquigarrow \hat{\varsigma}' \text{ and } \hat{\varsigma} \in \hat{S}\}$$

$$\mathrm{lfp}(\hat{f})$$

$$\mathrm{lfp}(\hat{f}) = \{\hat{\varsigma}' \mid \hat{\varsigma}_0 \rightsquigarrow^* \hat{\varsigma}'\}$$

$$\hat{V} = \{\hat{\varsigma}' \mid \hat{\varsigma}_0 \rightsquigarrow^* \hat{\varsigma}'\}$$

Case study: CPS and 0CFA

$$v \in \mathsf{Var} \text{ is a set of variables}$$

$$lam \in \mathsf{Lam} ::= (\lambda\ (v_1 \ldots v_n)\ call)$$

$$f, æ \in \mathsf{AExp} ::= v \mid lam$$

$$call \in \mathsf{Call} ::= (f\ æ_1 \ldots æ_n)$$

$$\Sigma = \mathsf{Call} \times Env$$

$$Env = \mathsf{Var} \to Clo$$

$$Clo = \mathsf{Lam} \times Env$$

$$\varsigma \in \Sigma = \mathsf{Call} \times Env$$

$$\rho \in Env = \mathsf{Var} \to Clo$$

$$clo \in Clo = \mathsf{Lam} \times Env$$

$$\mathcal{A} : \mathsf{AExp} \times Env \to Clo$$

$$\mathcal{A} : \mathsf{AExp} \times Env \rightarrow Clo$$

$$\mathcal{A}(v, \rho) = \rho(v)$$

$$\mathcal{A}(lam, \rho) = (lam, \rho)$$

$$\mathcal{I} : \mathsf{Call} \to \Sigma$$

$$\mathcal{I} : \mathsf{Call} \to \Sigma$$

$$\mathcal{I}(call) = (call, [])$$

$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \rho) \Rightarrow$$

$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \rho) \Rightarrow (\quad\quad, \quad\quad), \text{ where}$$

$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \rho) \Rightarrow (\quad\quad, \quad\quad), \text{ where}$$

$$f$$

$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \rho) \Rightarrow (\quad\quad , \quad\quad), \text{ where}$$
$$= \mathcal{A}(f, \quad)$$

$$([\![(f\ æ_1 \ldots æ_n)]\!], \rho) \Rightarrow (\quad\quad , \quad\quad \ddots), \text{ where}$$
$$= \mathcal{A}(f, \rho)$$

$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \rho) \Rightarrow (\quad\quad , \quad\quad), \text{ where}$$

$$(\llbracket (\lambda\ (v_1 \ldots v_n)\ \ call) \rrbracket, \rho') = \mathcal{A}(f, \rho)$$

$$([\![(f\ æ_1 \ldots æ_n)]\!], \rho) \Rightarrow (call, \quad \ddots), \text{ where}$$

$$([\![(\lambda\ (v_1 \ldots v_n)\ call)]\!], \rho') = \mathcal{A}(f, \rho)$$

$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \rho) \Rightarrow (call, \quad), \text{ where}$$

$$(\llbracket (\lambda\ (v_1 \ldots v_n)\ call) \rrbracket, \rho') = \mathcal{A}(f, \rho)$$

$$\rho'$$

$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \rho) \Rightarrow (call, \quad \ddots), \text{ where}$$

$$(\llbracket (\lambda\ (v_1 \ldots v_n)\ call) \rrbracket, \rho') = \mathcal{A}(f, \rho)$$

$$\rho'[v_i \mapsto \mathcal{A}(æ_i, \rho)]$$

$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \rho) \Rightarrow (call, \quad \ddots), \text{ where}$$

$$(\llbracket (\lambda\ (v_1 \ldots v_n)\ call) \rrbracket, \rho') = \mathcal{A}(f, \rho)$$

$$\rho'' = \rho'[v_i \mapsto \mathcal{A}(æ_i, \rho)]$$

$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \rho) \Rightarrow (call, \rho'')\text{, where}$$

$$(\llbracket (\lambda\ (v_1 \ldots v_n)\ call) \rrbracket, \rho') = \mathcal{A}(f, \rho)$$

$$\rho'' = \rho'[v_i \mapsto \mathcal{A}(æ_i, \rho)]$$

Structural abstraction?

$$\varsigma \in \Sigma = \mathsf{Call} \times Env$$

$$\rho \in Env = \mathsf{Var} \to Clo$$

$$clo \in Clo = \mathsf{Lam} \times Env$$

$$\hat{\varsigma} \in \widehat{\Sigma} = \mathsf{Call} \times \widehat{Env}$$

$$\hat{\rho} \in \widehat{Env} = \mathsf{Var} \to \widehat{Clo}$$

$$\widehat{clo} \in \widehat{Clo} = \mathsf{Lam} \times \widehat{Env}$$

$$\varsigma \in \Sigma = \mathsf{Call} \times Env$$

$$\rho \in Env = \mathsf{Var} \to Clo$$

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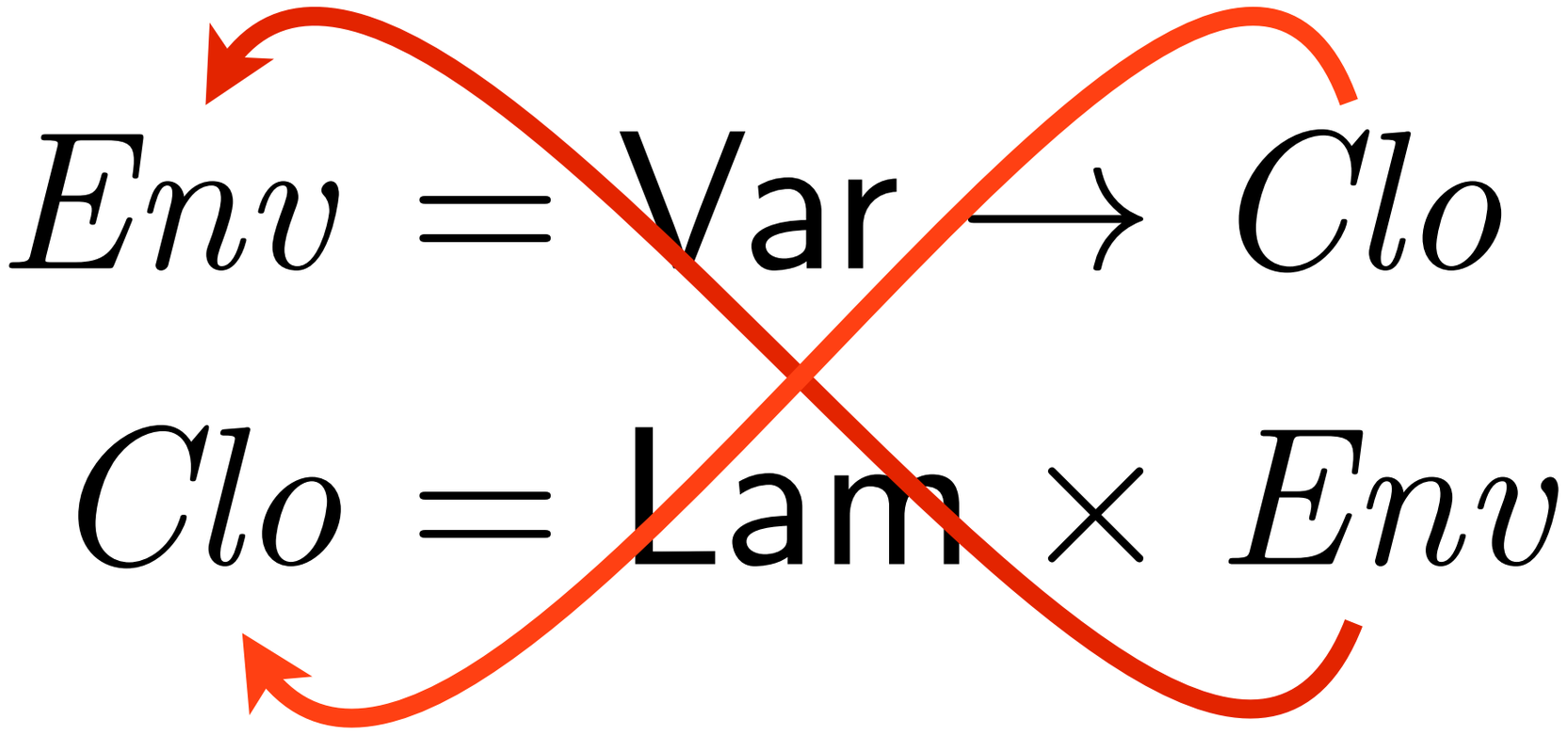
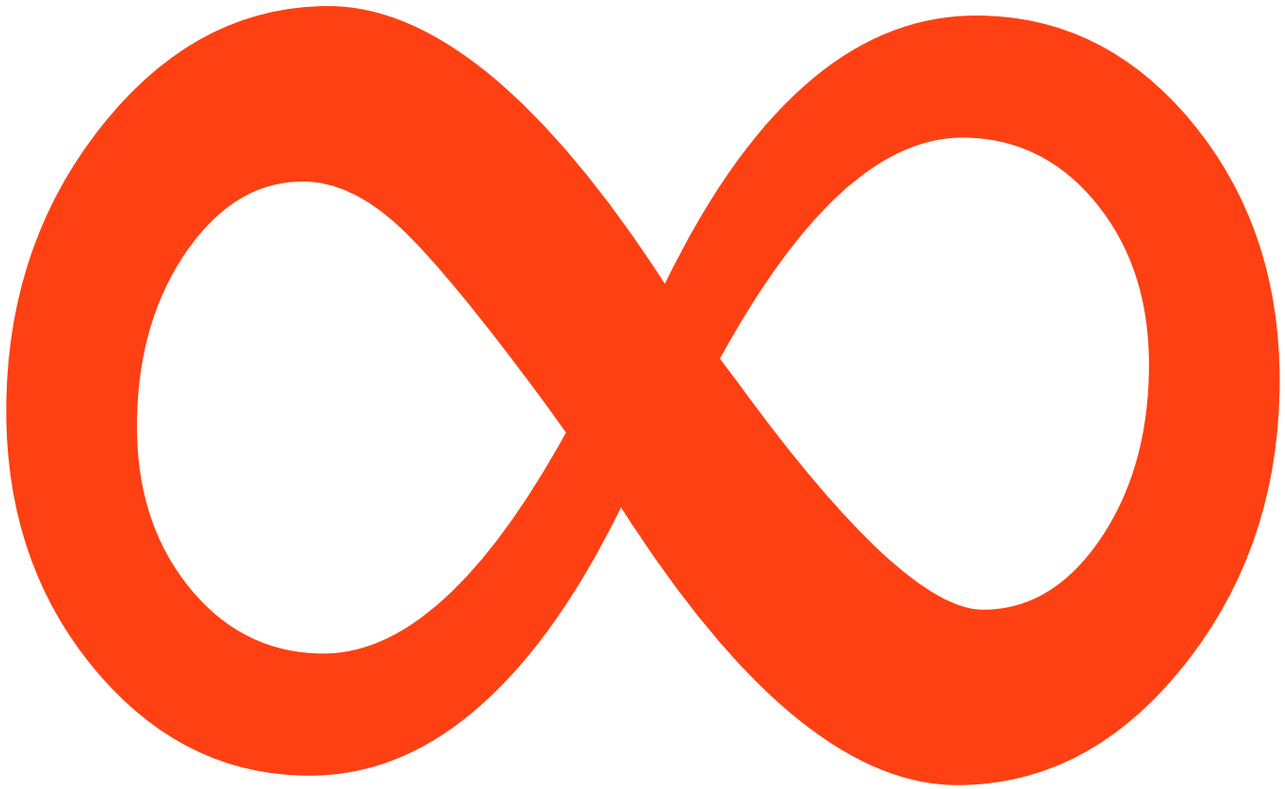
$$Env = \mathsf{Var} \to Clo$$

$$Clo = \mathsf{Lam} \times Env$$


$Env = \mathsf{Var} \to Clo$
$Clo = \mathsf{Lam} \times Env$



How to cut the knot?

Scott & Strachey, 1966

Store-passing transform.

$$\varsigma \in \Sigma = \mathsf{Call} \times Env$$

$$\rho \in Env = \mathsf{Var} \to Clo$$

$$clo \in Clo = \mathsf{Lam} \times Env$$

$$\varsigma \in \Sigma = \mathsf{Call} \times Env$$

$$\rho \in Env = \mathsf{Var} \to Clo$$

$$clo \in Clo = \mathsf{Lam} \times Env$$

$$\varsigma \in \Sigma = \mathsf{Call} \times Env \times Store$$

$$\rho \in Env = \mathsf{Var} \to Clo$$

$$clo \in Clo = \mathsf{Lam} \times Env$$

$$\varsigma \in \Sigma = \mathsf{Call} \times Env \times Store$$

$$\rho \in Env = \mathsf{Var} \to Clo$$

$$clo \in Clo = \mathsf{Lam} \times Env$$

$$\sigma \in Store = Addr \to$$

$a \in Addr$ is an infinite set.

$$\varsigma \in \Sigma = \mathsf{Call} \times Env \times Store$$

$$\rho \in Env = \mathsf{Var} \to$$

$$clo \in Clo = \mathsf{Lam} \times Env$$

$$\sigma \in Store = Addr \to Clo$$

$a \in Addr$ is an infinite set.

$$\varsigma \in \Sigma = \mathsf{Call} \times Env \times Store$$

$$\rho \in Env = \mathsf{Var} \to Addr$$

$$clo \in Clo = \mathsf{Lam} \times Env$$

$$\sigma \in Store = Addr \to Clo$$

$$a \in Addr \text{ is an infinite set.}$$

$$\mathcal{A} : \mathsf{AExp} \times Env \rightarrow Clo$$

$$\mathcal{A} : \mathsf{AExp} \times \mathit{Env} \times \mathit{Store} \to \mathit{Clo}$$

$$\mathcal{A} : \mathsf{AExp} \times \mathit{Env} \times \mathit{Store} \to \mathit{Clo}$$

$$\mathcal{A}(v, \rho, \sigma) = \sigma(\rho(v))$$

$$\mathcal{A}(\mathit{lam}, \rho, \sigma) = (\mathit{lam}, \rho)$$

$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \rho) \Rightarrow (call, \rho''), \text{ where}$$

$$(\llbracket (\lambda\ (v_1 \ldots v_n)\ call) \rrbracket, \rho') = \mathcal{A}(f, \rho)$$

$$\rho'' = \rho'[v_i \mapsto \mathcal{A}(æ_i, \rho)]$$

$$(\llbracket(f\ æ_1 \ldots æ_n)\rrbracket, \rho\ , \sigma) \Rightarrow (call, \rho''\ ), \text{ where}$$

$$(\llbracket(\lambda\ (v_1 \ldots v_n)\ call)\rrbracket, \rho') = \mathcal{A}(f, \rho)$$

$$\rho'' = \rho'[v_i \mapsto \mathcal{A}(æ_i, \rho)]$$

$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \rho\ , \sigma) \Rightarrow (call, \rho'', \sigma'), \text{ where}$$

$$(\llbracket (\lambda\ (v_1 \ldots v_n)\ call) \rrbracket, \rho') = \mathcal{A}(f, \rho)$$

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$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \rho\ , \sigma) \Rightarrow (call, \rho'', \sigma'), \text{ where}$$

$$(\llbracket (\lambda\ (v_1 \ldots v_n)\ call) \rrbracket, \rho') = \mathcal{A}(f, \rho, \sigma)$$

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$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \rho\ , \sigma) \Rightarrow (call, \rho'', \sigma'), \text{ where}$$

$$(\llbracket (\lambda\ (v_1 \ldots v_n)\ call) \rrbracket, \rho') = \mathcal{A}(f, \rho, \sigma)$$

$$\rho'' = \rho'[v_i \mapsto \mathcal{A}(æ_i, \rho\ , \sigma)]$$

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$$(\llbracket (\lambda\ (v_1 \ldots v_n)\ call) \rrbracket, \rho') = \mathcal{A}(f, \rho, \sigma)$$

$$\rho'' = \rho'[v_i \mapsto a_i]$$

$$\mathcal{A}(æ_i, \rho\ , \sigma)]$$

$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \rho\ , \sigma) \Rightarrow (call, \rho'', \sigma'), \text{ where}$$

$$(\llbracket (\lambda\ (v_1 \ldots v_n)\ call) \rrbracket, \rho') = \mathcal{A}(f, \rho, \sigma)$$

$$\rho'' = \rho'[v_i \mapsto a_i]$$

$$\mathcal{A}(æ_i, \rho\ , \sigma)]$$

$$a_i = alloc(v_i, \sigma)$$

$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \rho\ , \sigma) \Rightarrow (call, \rho'', \sigma'), \text{ where}$$

$$(\llbracket (\lambda\ (v_1 \ldots v_n)\ call) \rrbracket, \rho') = \mathcal{A}(f, \rho, \sigma)$$

$$\rho'' = \rho'[v_i \mapsto a_i]$$

$$\sigma' = \sigma[a_i \mapsto \mathcal{A}(æ_i, \rho\ , \sigma)]$$

$$a_i = alloc(v_i, \sigma)$$

Abstraction

$$\varsigma \in \Sigma = \mathsf{Call} \times Env \times Store$$

$$\rho \in Env = \mathsf{Var} \to Addr$$

$$clo \in Clo = \mathsf{Lam} \times Env$$

$$\sigma \in Store = Addr \to Clo$$

$a \in Addr$ is an infinite set.

$$\varsigma \in \Sigma = \mathsf{Call} \times Env \times Store$$

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$$clo \in Clo = \mathsf{Lam} \times Env$$

$$\sigma \in Store = Addr \to Clo$$

$a \in Addr$ is a finite set.

$$\varsigma \in \Sigma = \mathsf{Call} \times Env \times Store$$

$$\rho \in Env = \mathsf{Var} \to Addr$$

$$clo \in Clo = \mathsf{Lam} \times Env$$

$$\sigma \in Store = Addr \to \mathcal{P}(\ Clo\ )$$

$$a \in Addr \text{ is a finite set.}$$

$$\hat{\varsigma} \in \widehat{\Sigma} = \mathsf{Call} \times \widehat{Env} \times \widehat{Store}$$

$$\hat{\rho} \in \widehat{Env} = \mathsf{Var} \to \widehat{Addr}$$

$$\widehat{clo} \in \widehat{Clo} = \mathsf{Lam} \times \widehat{Env}$$

$$\hat{\sigma} \in \widehat{Store} = \widehat{Addr} \to \mathcal{P}(\ \widehat{Clo}\ )$$

$$\hat{a} \in \widehat{Addr} \text{ is a finite set.}$$

$$\widehat{Addr} : Addr \to Addr$$

$$\alpha : \sum \to \sum$$

$$\alpha(call, \rho, \sigma) =$$

$$\alpha(call, \rho, \sigma) =$$

$$\alpha(call, \rho, \sigma) = (\quad , \quad , \quad)$$

$$\alpha(call, \rho, \sigma) = (call, \quad \rho \ , \quad \sigma \ )$$

$$\alpha(call, \rho, \sigma) = (call, \alpha(\rho), \quad \sigma \ )$$

$$\alpha(call, \rho, \sigma) = (call, \alpha(\rho), \alpha(\sigma))$$

$$\alpha(call, \rho, \sigma) = (call, \alpha(\rho), \alpha(\sigma))$$

$$\alpha(\rho) = \lambda v.\alpha(\rho(v))$$

$$\alpha(\sigma) = \lambda \hat{a}. \bigsqcup_{\alpha(a)=\hat{a}} \{\alpha(\sigma(a))\}$$

$$\alpha(lam, \rho) = \{(lam, \alpha(\rho))\}$$

$$\alpha(a) \text{ is to be continued...}$$

$$\alpha(call, \rho, \sigma) = (call, \alpha(\rho), \alpha(\sigma))$$

$$\alpha(\rho) = \lambda v.\, \alpha(\rho(v))$$

$$\alpha(\sigma) = \lambda \hat{a}. \bigsqcup \{\alpha(\sigma(a))\}$$
$$\alpha(a)=\hat{a}$$

$$\alpha(lam, \rho) = \{(lam, \alpha(\rho))\}$$

$\alpha(a)$ is to be continued...

$$(\llbracket(f\ æ_1 \ldots æ_n)\rrbracket, \rho, \sigma) \Rightarrow (call, \rho'', \sigma'), \text{ where}$$

$$(\llbracket(\lambda\ (v_1 \ldots v_n)\ call)\rrbracket, \rho') = \mathcal{A}(f, \rho, \sigma)$$

$$\rho'' = \rho'[v_i \mapsto a_i]$$

$$\sigma' = \sigma\left[a_i \mapsto \mathcal{A}(æ_i, \rho, \sigma)\right]$$

$$a_i = alloc(v_i, \sigma)$$

$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \hat{\rho}, \hat{\sigma}) \Rightarrow (call, \hat{\rho}'', \hat{\sigma}'), \text{ where}$$

$$(\llbracket (\lambda\ (v_1 \ldots v_n)\ call) \rrbracket, \hat{\rho}') = \hat{\mathcal{A}}(f, \hat{\rho}, \hat{\sigma})$$

$$\hat{\rho}'' = \hat{\rho}'[v_i \mapsto \hat{a}_i]$$

$$\hat{\sigma}' = \hat{\sigma}\,[\hat{a}_i \mapsto \hat{\mathcal{A}}(æ_i, \hat{\rho}, \hat{\sigma})]$$

$$\hat{a}_i = \widehat{alloc}(v_i, \hat{\sigma})$$

$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \hat{\rho}, \hat{\sigma}) \rightsquigarrow (call, \hat{\rho}'', \hat{\sigma}'), \text{ where}$$

$$(\llbracket (\lambda\ (v_1 \ldots v_n)\ call) \rrbracket, \hat{\rho}') = \hat{\mathcal{A}}(f, \hat{\rho}, \hat{\sigma})$$

$$\hat{\rho}'' = \hat{\rho}'[v_i \mapsto \hat{a}_i]$$

$$\hat{\sigma}' = \hat{\sigma}\,[\hat{a}_i \mapsto \hat{\mathcal{A}}(æ_i, \hat{\rho}, \hat{\sigma})]$$

$$\hat{a}_i = \widehat{alloc}(v_i, \hat{\sigma})$$

$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \hat{\rho}, \hat{\sigma}) \rightsquigarrow (call, \hat{\rho}'', \hat{\sigma}'), \text{ where}$$

$$(\llbracket (\lambda\ (v_1 \ldots v_n)\ call) \rrbracket, \hat{\rho}') = \hat{\mathcal{A}}(f, \hat{\rho}, \hat{\sigma})$$

$$\hat{\rho}'' = \hat{\rho}'[v_i \mapsto \hat{a}_i]$$

$$\hat{\sigma}' = \hat{\sigma} \sqcup [\hat{a}_i \mapsto \hat{\mathcal{A}}(æ_i, \hat{\rho}, \hat{\sigma})]$$

$$\hat{a}_i = \widehat{alloc}(v_i, \hat{\sigma})$$

$$(\llbracket (f\ æ_1 \ldots æ_n) \rrbracket, \hat{\rho}, \hat{\sigma}) \rightsquigarrow (call, \hat{\rho}'', \hat{\sigma}'), \text{ where}$$

$$(\llbracket (\lambda\ (v_1 \ldots v_n)\ call) \rrbracket, \hat{\rho}') \in \hat{\mathcal{A}}(f, \hat{\rho}, \hat{\sigma})$$

$$\hat{\rho}'' = \hat{\rho}'[v_i \mapsto \hat{a}_i]$$

$$\hat{\sigma}' = \hat{\sigma} \sqcup [\hat{a}_i \mapsto \hat{\mathcal{A}}(æ_i, \hat{\rho}, \hat{\sigma})]$$

$$\hat{a}_i = \widehat{alloc}(v_i, \hat{\sigma})$$

Allocation

$$(\llbracket(f\ æ_1 \ldots æ_n)\rrbracket, \hat{\rho}, \hat{\sigma}) \Rightarrow (call, \hat{\rho}'', \hat{\sigma}'), \text{ where}$$

$$(\llbracket(\lambda\ (v_1 \ldots v_n)\ call)\rrbracket, \hat{\rho}') = \hat{\mathcal{A}}(f, \hat{\rho}, \hat{\sigma})$$

$$\hat{\rho}'' = \hat{\rho}'[v_i \mapsto \hat{a}_i]$$

$$\hat{\sigma}' = \hat{\sigma} \sqcup [\hat{a}_i \mapsto \hat{\mathcal{A}}(æ_i, \hat{\rho}, \hat{\sigma})]$$

$$\hat{a}_i = \widehat{alloc}(v_i, \hat{\sigma})$$

$$(\llbracket(f\ æ_1 \ldots æ_n)\rrbracket, \hat{\rho}, \hat{\sigma}) \Rightarrow (call, \hat{\rho}'', \hat{\sigma}'), \text{ where}$$

$$(\llbracket(\lambda\ (v_1 \ldots v_n)\ call)\rrbracket, \hat{\rho}') = \hat{\mathcal{A}}(f, \hat{\rho}, \hat{\sigma})$$

$$\hat{\rho}'' = \hat{\rho}'[v_i \mapsto \hat{a}_i]$$

$$\hat{\sigma}' = \hat{\sigma} \sqcup [\hat{a}_i \mapsto \hat{\mathcal{A}}(æ_i, \hat{\rho}, \hat{\sigma})]$$

$$\hat{a}_i = \widehat{alloc}(v_i, \hat{\sigma})$$

Determines polyvariance

0CFA

$$Addr = \mathsf{Var}$$

$$\widehat{alloc}(v, \hat{\sigma}) =$$

$$\widehat{alloc}(v, \hat{\sigma}) = v$$

Lessons?

Step 1: Cut recursion.

Step 2: Structurally abstract.

Case study: ANF

$$
\begin{aligned}
e \in \mathsf{Exp} &::= \texttt{(let ((}v\ call\texttt{))}\ e\texttt{)} \\
&\mid\ call \\
&\mid\ æ \\
f, æ \in \mathsf{AExp} &::= v \mid lam \\
lam \in \mathsf{Lam} &::= (\lambda\ (v)\ e) \\
call \in \mathsf{Call} &::= (f\ æ) \\
v \in \mathsf{Var} &\text{ is a set of identifiers}
\end{aligned}
$$

ČESK

CESK*

CESK*

$$\Sigma = \mathsf{Exp} \times Env \times Store \times Kont$$

C E S K

$$\Sigma = \mathsf{Exp} \times Env \times Store \times Kont$$

state-space

$$\Sigma = \mathsf{Exp} \times Env \times Store \times Kont$$

$$\Sigma = \mathsf{Exp} \times Env \times Store \times Kont$$

$$Store = Addr \to Clo$$

$$Env = \mathsf{Var} \to Addr$$

$$Clo = \mathsf{Lam} \times Env$$

$$\kappa \in Kont ::= \mathbf{letk}(v, e, \rho, \kappa)$$

$$\mid \ \mathbf{halt}$$

$$\hat{\Sigma} = \mathsf{Exp} \times \hat{Env} \times \hat{Store} \times \hat{Kont}$$

$$\hat{Store} = \hat{Addr} \to \hat{Clo}$$

$$Env = \mathsf{Var} \to Addr$$

$$\hat{Clo} = \mathsf{Lam} \times \hat{Env}$$

$$\hat{\kappa} \in \hat{Kont} ::= \mathbf{letk}(v, e, \hat{\rho}, \hat{\kappa}) \mid \mathbf{halt}$$

$$\hat{\Sigma} = \mathsf{Exp} \times \hat{Env} \times \hat{Store} \times \hat{Kont}$$

$$\hat{Store} = \hat{Addr} \to \hat{Clo}$$

$$Env = \mathsf{Var} \to Addr$$

$$\hat{Clo} = \mathsf{Lam} \times \hat{Env}$$

$$\hat{\kappa} \in \hat{Kont} ::= \mathbf{letk}(v, e, \hat{\rho}, \hat{\kappa}) \mid \mathbf{halt}$$

$$\kappa \in Kont ::= \mathbf{letk}(v, e, \rho, \kappa)$$

$$\kappa \in Kont ::= \mathbf{letk}(v, e, \rho, \kappa)$$

$$\kappa \in Kont ::= \mathbf{letk}(v, e, \rho, \kappa)$$

$$\hat{\kappa} \in \widehat{Kont} ::= \mathbf{letk}(v, e, \hat{\rho}, \hat{\kappa})$$

Store-allocate $Kont$

$$Store = Addr \to Clo$$

$$Store = Addr \to Clo + Kont$$

$$\Sigma = \mathsf{Exp} \times Env \times Store \times Kont$$

$$\Sigma = \mathsf{Exp} \times Env \times Store \times Addr$$

$$\kappa \in Kont ::= \mathbf{letk}(v, e, \rho, \kappa)$$

$$\kappa \in Kont ::= \mathbf{letk}(v, e, \rho, a)$$

$$(\llbracket (f\ æ) \rrbracket, \rho, \sigma, a_\kappa) \Rightarrow (e, \rho', \sigma', a_\kappa), \text{ where}$$

$$(\llbracket (\lambda\ (v_1 \ldots v_n)\ call) \rrbracket, \rho') = \mathcal{A}(f, \rho, \sigma)$$

$$\rho'' = \rho'[v_i \mapsto a_i]$$

$$\sigma' = \sigma[a_i \mapsto \mathcal{A}(æ_i, \rho, \sigma)]$$

$$a_i = alloc(v_i, \ldots)$$

$$(æ, \rho, \sigma, a_\kappa) \Rightarrow (e, \rho'', \sigma', a'_\kappa), \text{ where}$$

$$\mathbf{letk}(v, e, \rho', a'_\kappa) = \sigma(a_k)$$

$$\rho'' = \rho'[v \mapsto a]$$

$$\sigma' = \sigma[a \mapsto \mathcal{A}(æ, \rho, \sigma)]$$

$$a = alloc(v, \ldots)$$

$$([\![\texttt{(let ((}v\ call\texttt{))}\ e\texttt{)}]\!], \rho, \sigma, a_\kappa) \Rightarrow (call, \rho, \sigma', a'_\kappa), \text{ where}$$

$$\sigma' = \sigma[a'_\kappa \mapsto \mathbf{letk}(v, e, \rho, a_\kappa)]$$

$$a'_\kappa = alloc(\ldots)$$

Structural abstraction

$$\varsigma \in \Sigma = \mathsf{Exp} \times Env \times Store \times Addr$$

$$\rho \in Env = \mathsf{Var} \rightharpoonup Addr$$

$$\sigma \in Store = Addr \rightharpoonup Clo + Kont$$

$$clo \in Clo = \mathsf{Lam} \times Env$$

$$\kappa \in Kont ::= \mathbf{letk}(v, e, \rho, a) \mid \mathbf{halt}$$

$$a \in Addr \text{ is an infinite set}$$

$$\varsigma \in \Sigma = \mathsf{Exp} \times Env \times Store \times Addr$$

$$\rho \in Env = \mathsf{Var} \rightharpoonup Addr$$

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$$a \in Addr \text{ is a finite set}$$

$$\varsigma \in \Sigma = \mathsf{Exp} \times Env \times Store \times Addr$$

$$\rho \in Env = \mathsf{Var} \rightharpoonup Addr$$

$$\sigma \in Store = Addr \rightharpoonup \mathcal{P}(Clo + Kont)$$

$$clo \in Clo = \mathsf{Lam} \times Env$$

$$\kappa \in Kont ::= \mathbf{letk}(v, e, \rho, a)$$
$$\mid \quad \mathbf{halt}$$

$a \in Addr$ is a finite set

$$\widehat{\varsigma} \in \widehat{\Sigma} = \mathsf{Exp} \times \widehat{Env} \times \widehat{Store} \times \widehat{Addr}$$

$$\widehat{\rho} \in \widehat{Env} = \mathsf{Var} \rightharpoonup \widehat{Addr}$$

$$\widehat{\sigma} \in \widehat{Store} = \widehat{Addr} \rightharpoonup \mathcal{P}(\widehat{Clo} + \widehat{Kont})$$

$$\widehat{clo} \in \widehat{Clo} = \mathsf{Lam} \times \widehat{Env}$$

$$\widehat{\kappa} \in \widehat{Kont} ::= \mathbf{letk}(v, e, \widehat{\rho}, \widehat{a}) \mid \mathbf{halt}$$

$$\widehat{a} \in \widehat{Addr} \text{ is a finite set}$$

Pushdown?

$$\kappa \in Kont ::= \mathbf{letk}(v, e, \rho, \kappa)$$
$$\mid \quad \mathbf{halt}$$

$$\kappa \in Kont ::= \mathbf{letk}(v, e, \rho, \kappa)$$
$$\mid \quad \mathbf{halt}$$
$$::\cong \langle (v_1, e_1, \rho_1), \ldots, (v_n, e_n, \rho_n) \rangle$$

$$\kappa \in Kont = Frame^*$$

$$\phi \in Frame = \mathsf{Var} \times \mathsf{Exp} \times Env$$

$$c \in Conf = \mathsf{Exp} \times Env \times Store \times Kont$$

$$\rho \in Env = \mathsf{Var} \rightharpoonup Addr$$

$$\sigma \in Store = Addr \to Clo$$

$$clo \in Clo = \mathsf{Lam} \times Env$$

$$\kappa \in Kont = Frame^*$$

$$\phi \in Frame = \mathsf{Var} \times \mathsf{Exp} \times Env$$

$a \in Addr$ is an infinite set of addresses

$$\hat{c} \in \widehat{Conf} = \mathsf{Exp} \times \widehat{Env} \times \widehat{Store} \times \widehat{Kont}$$

$$\hat{\rho} \in \widehat{Env} = \mathsf{Var} \rightharpoonup \widehat{Addr}$$

$$\hat{\sigma} \in \widehat{Store} = \widehat{Addr} \to \mathcal{P}\left(\widehat{Clo}\right)$$

$$clo \in Clo = \mathsf{Lam} \times Env$$

$$\hat{\kappa} \in \widehat{Kont} = \widehat{Frame}^*$$

$$\hat{\phi} \in \widehat{Frame} = \mathsf{Var} \times \mathsf{Exp} \times \widehat{Env}$$

$\hat{a} \in \widehat{Addr}$ is a *finite* set of addresses

$$\hat{c} \in \widehat{Conf} = \mathsf{Exp} \times \widehat{Env} \times \widehat{Store} \times \widehat{Kont}$$

$$\hat{\rho} \in \widehat{Env} = \mathsf{Var} \rightharpoonup \widehat{Addr}$$

$$\hat{\sigma} \in \widehat{Store} = \widehat{Addr} \to \mathcal{P}\left(\widehat{Clo}\right)$$

$$clo \in Clo = \mathsf{Lam} \times Env$$

$$\hat{\kappa} \in \widehat{Kont} = \widehat{Frame}^*$$

$$\hat{\phi} \in \widehat{Frame} = \mathsf{Var} \times \mathsf{Exp} \times \widehat{Env}$$

$\hat{a} \in \widehat{Addr}$ is a *finite* set of addresses

It’s pushdown!

$$\hat{c} \in \widehat{Conf} = \mathsf{Exp} \times \widehat{Env} \times \widehat{Store} \times \widehat{Kont}$$

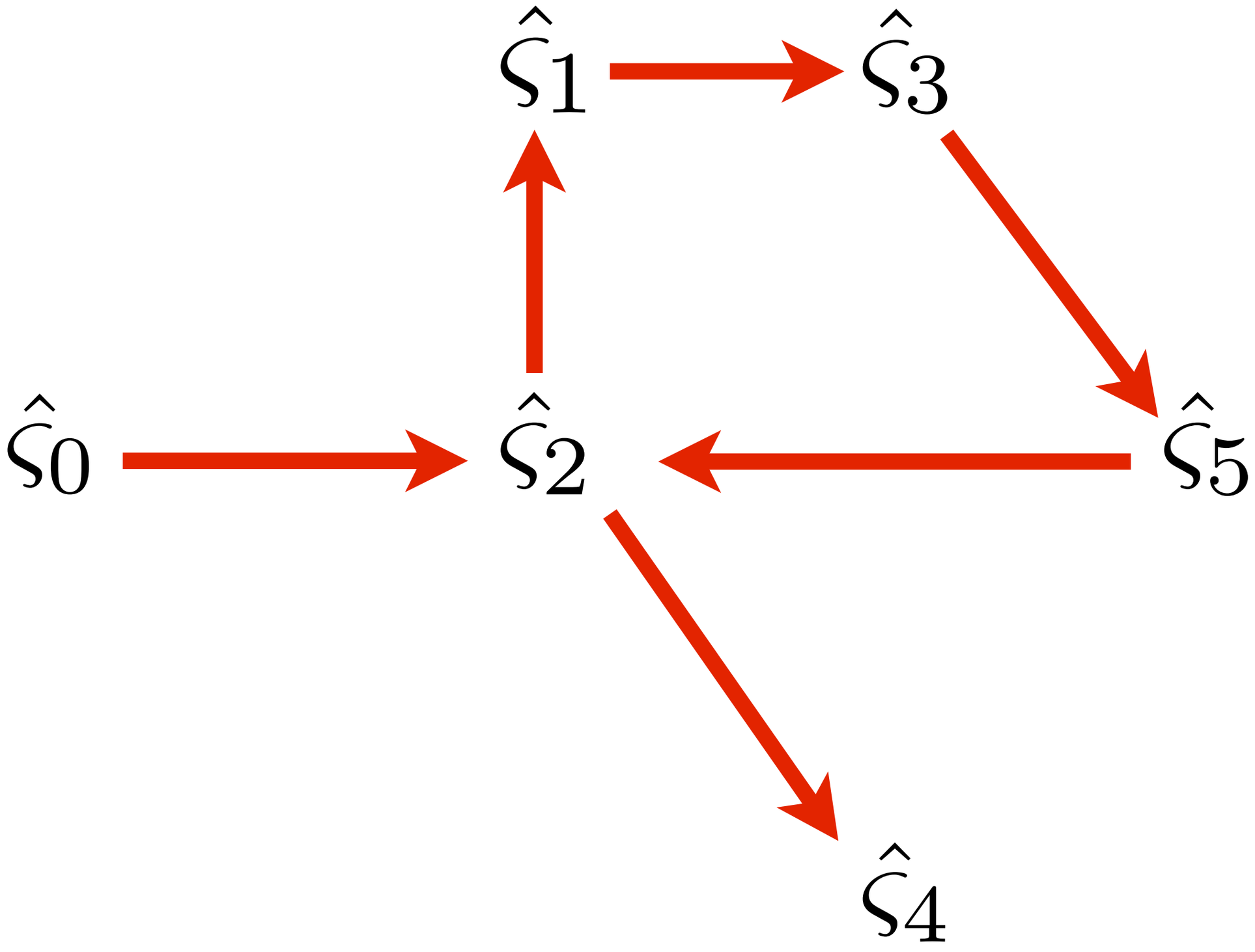
control states

stack

$$\hat{c} \in \widehat{Conf} = \mathsf{Exp} \times \widehat{Env} \times \widehat{Store} \times \widehat{Kont}$$

Control-state reachability

$\hat{\varsigma}_0$


$\hat{\varsigma}_1$
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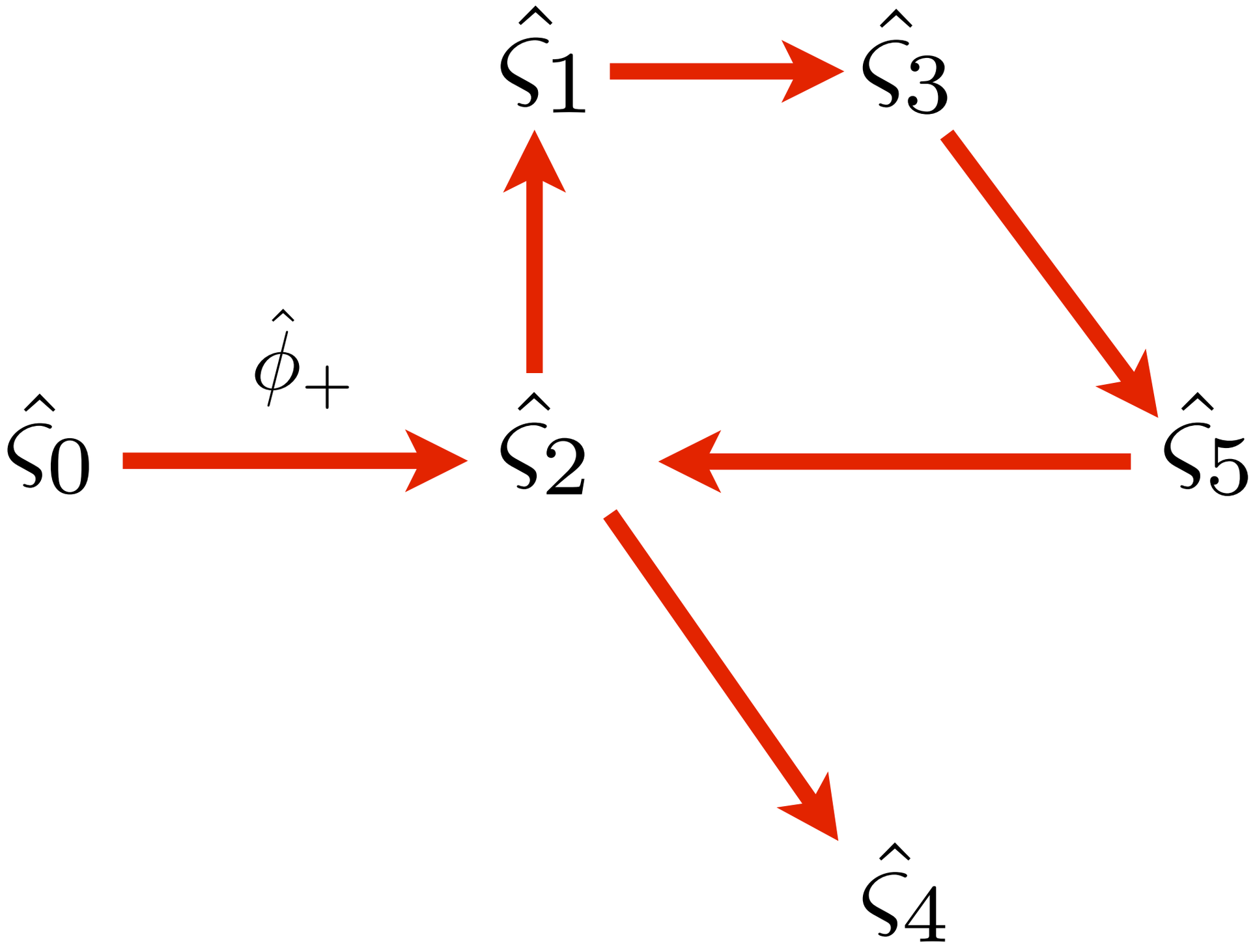
$\hat{\varsigma}_5$
$\hat{\varsigma}_4$


$\hat{\varsigma}_1$
$\hat{\varsigma}_3$
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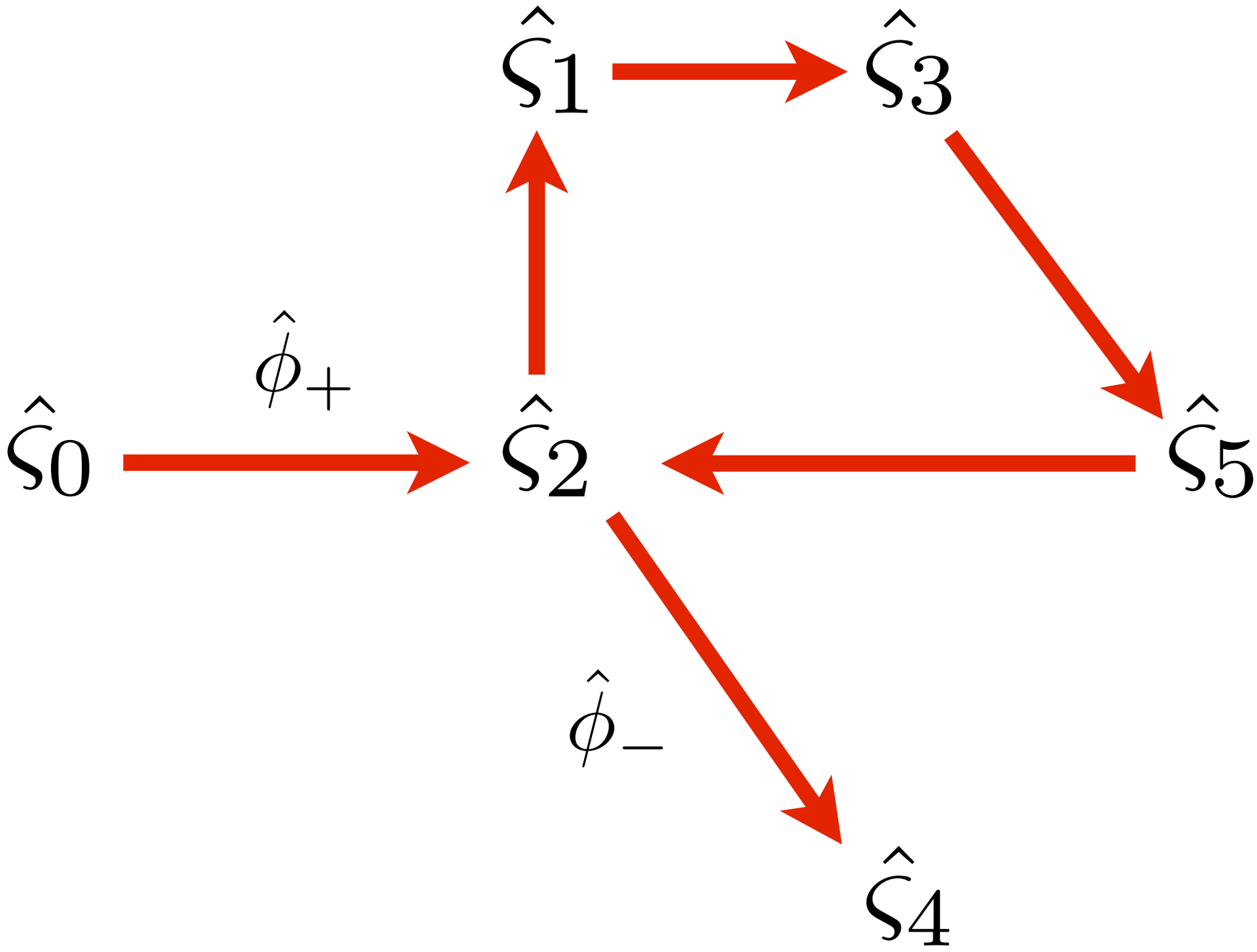
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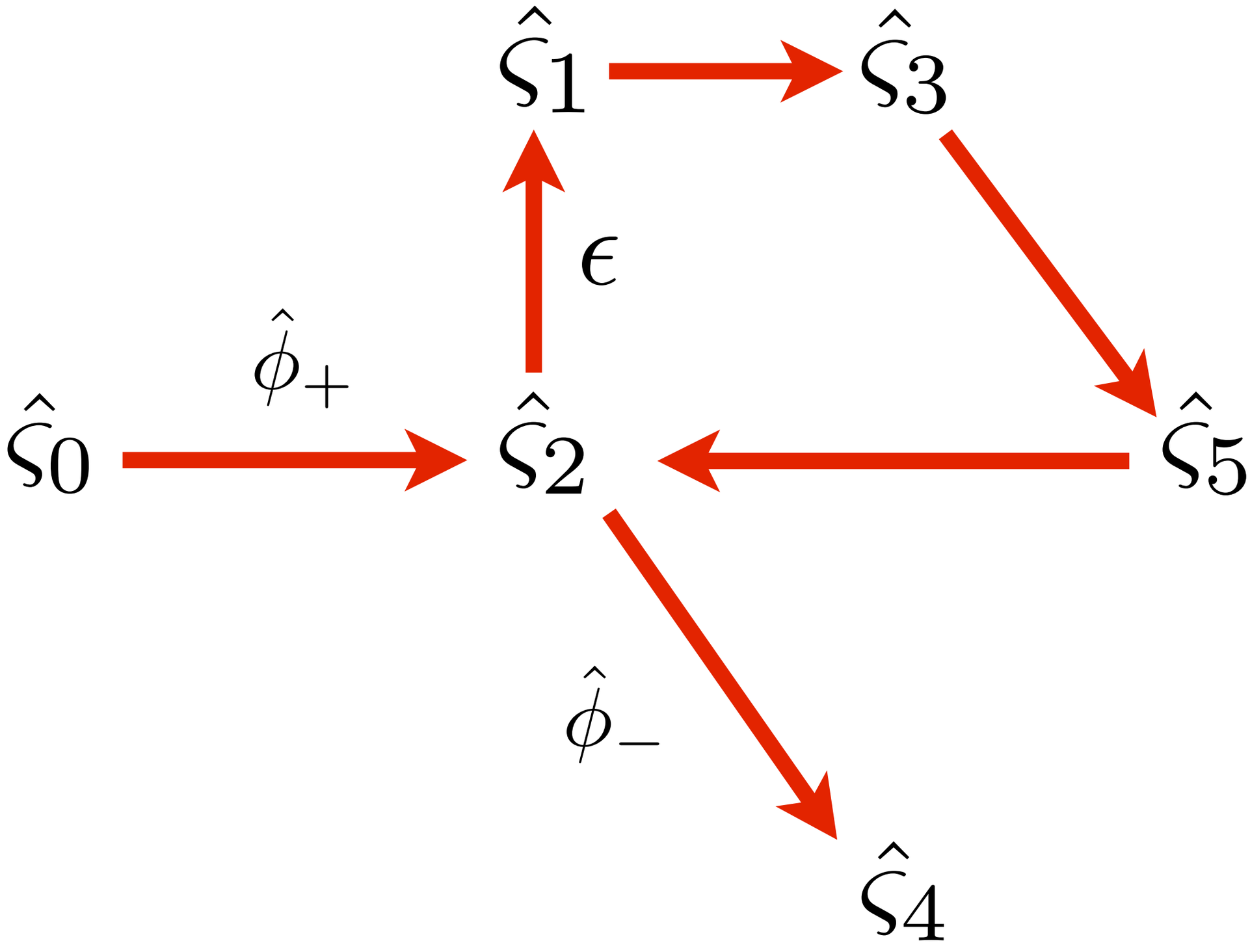
$\hat{\varsigma}_4$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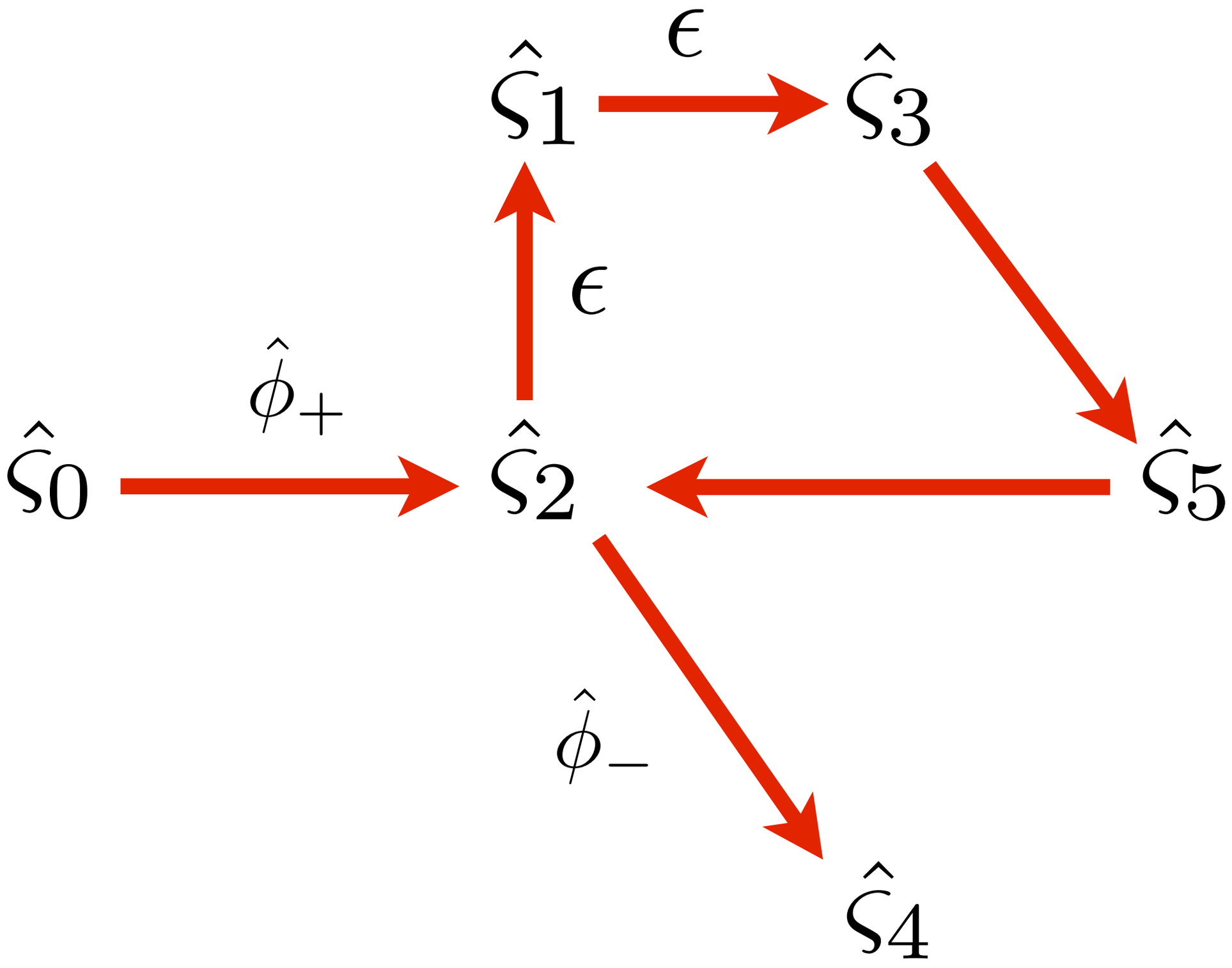
$\hat{\varsigma}_4$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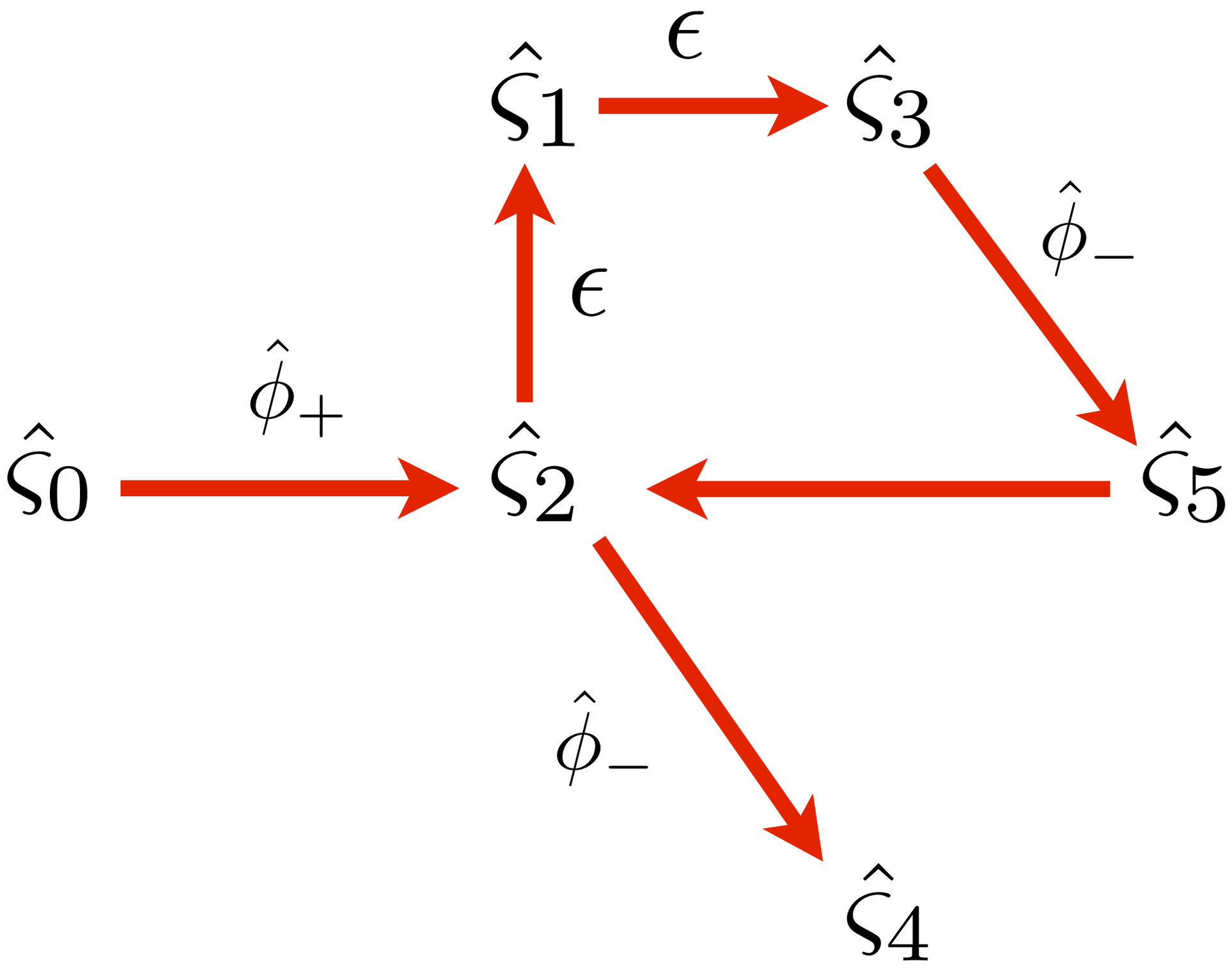
$\hat{\phi}_-$
$\hat{\varsigma}_4$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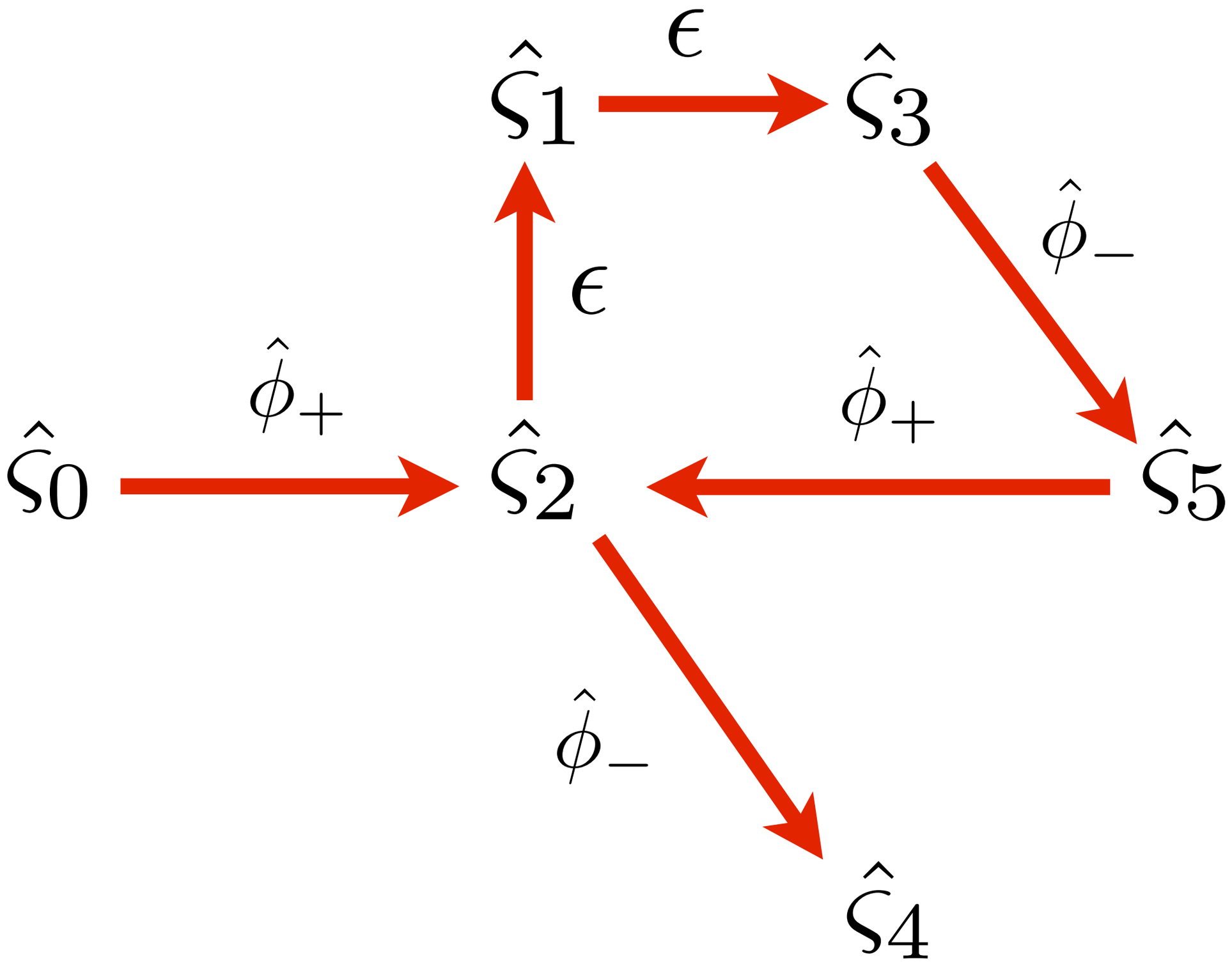
$\hat{\varsigma}_4$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$\hat{\varsigma}_4$

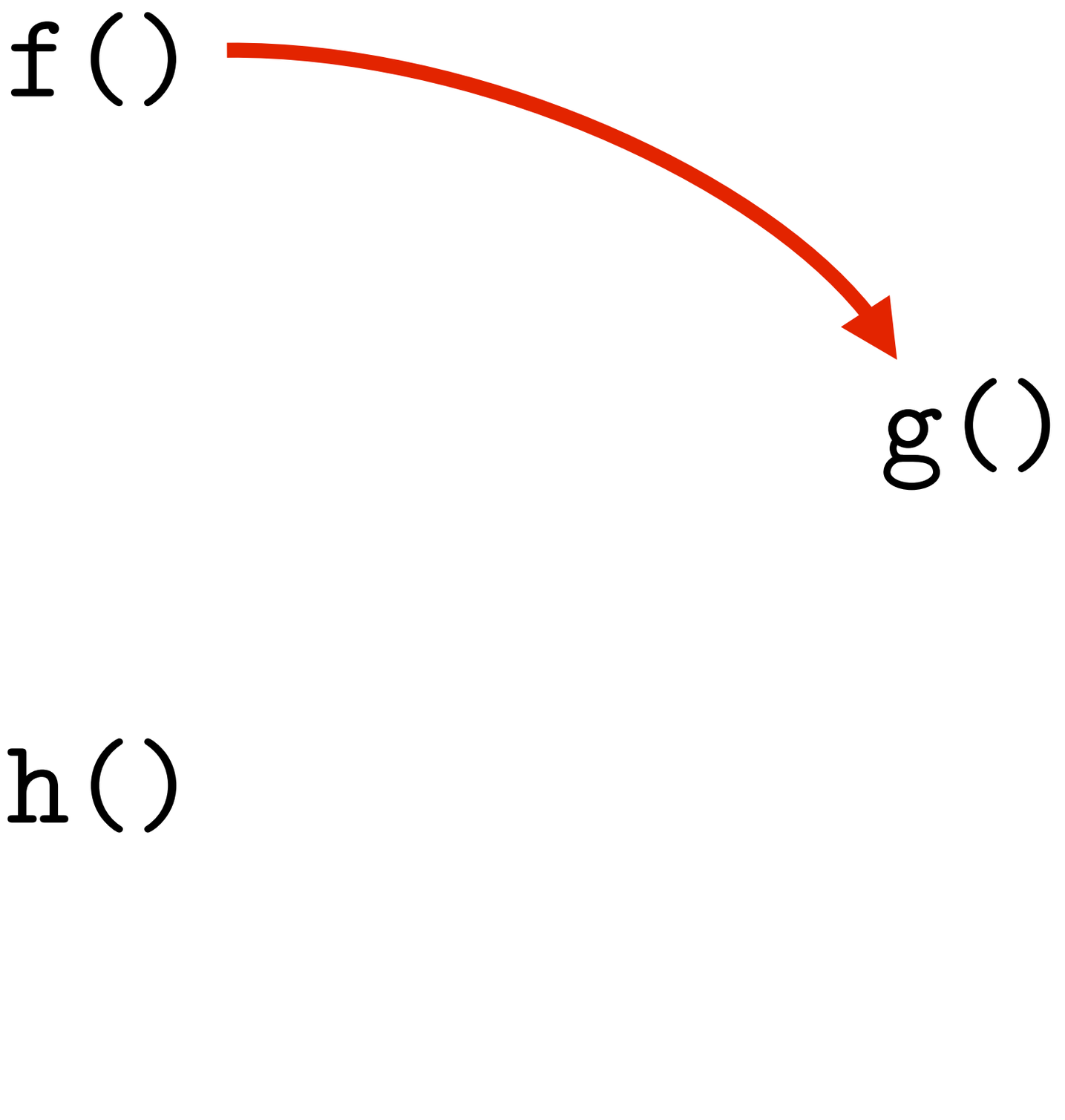
Advantage?

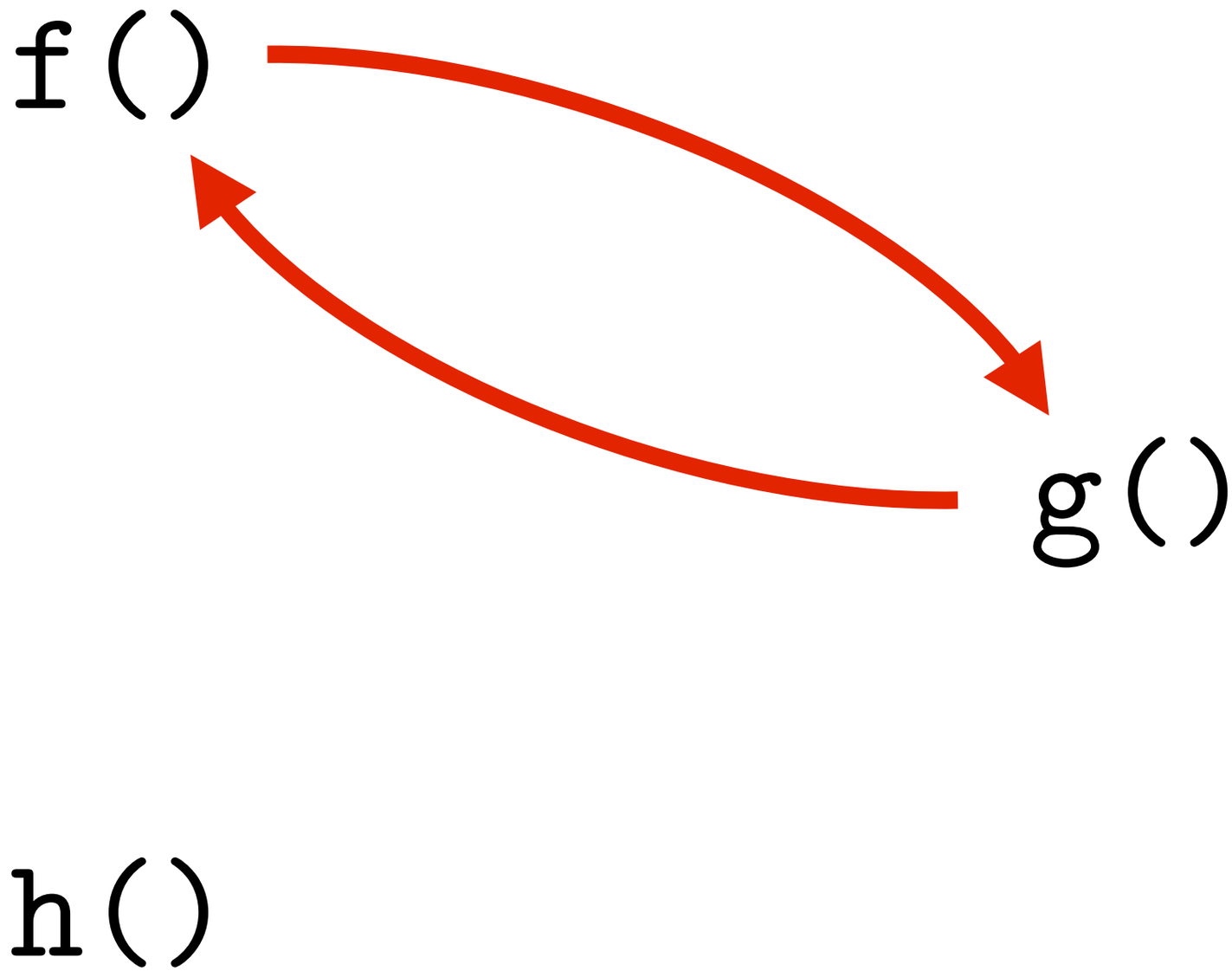
Matches returns to calls.

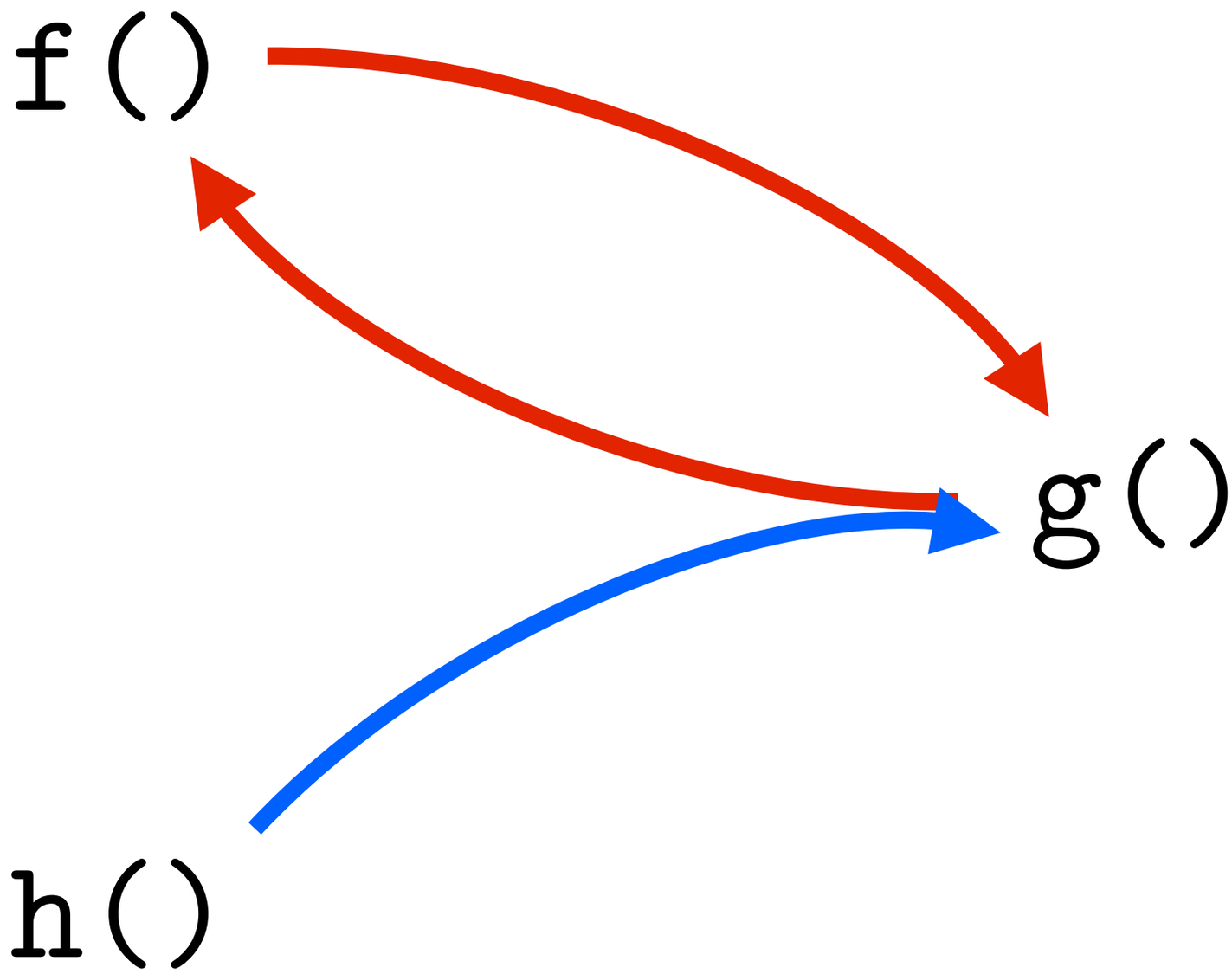
f()

g()

h()


f()
g()
h()


f()
g()
h()


f()
g()
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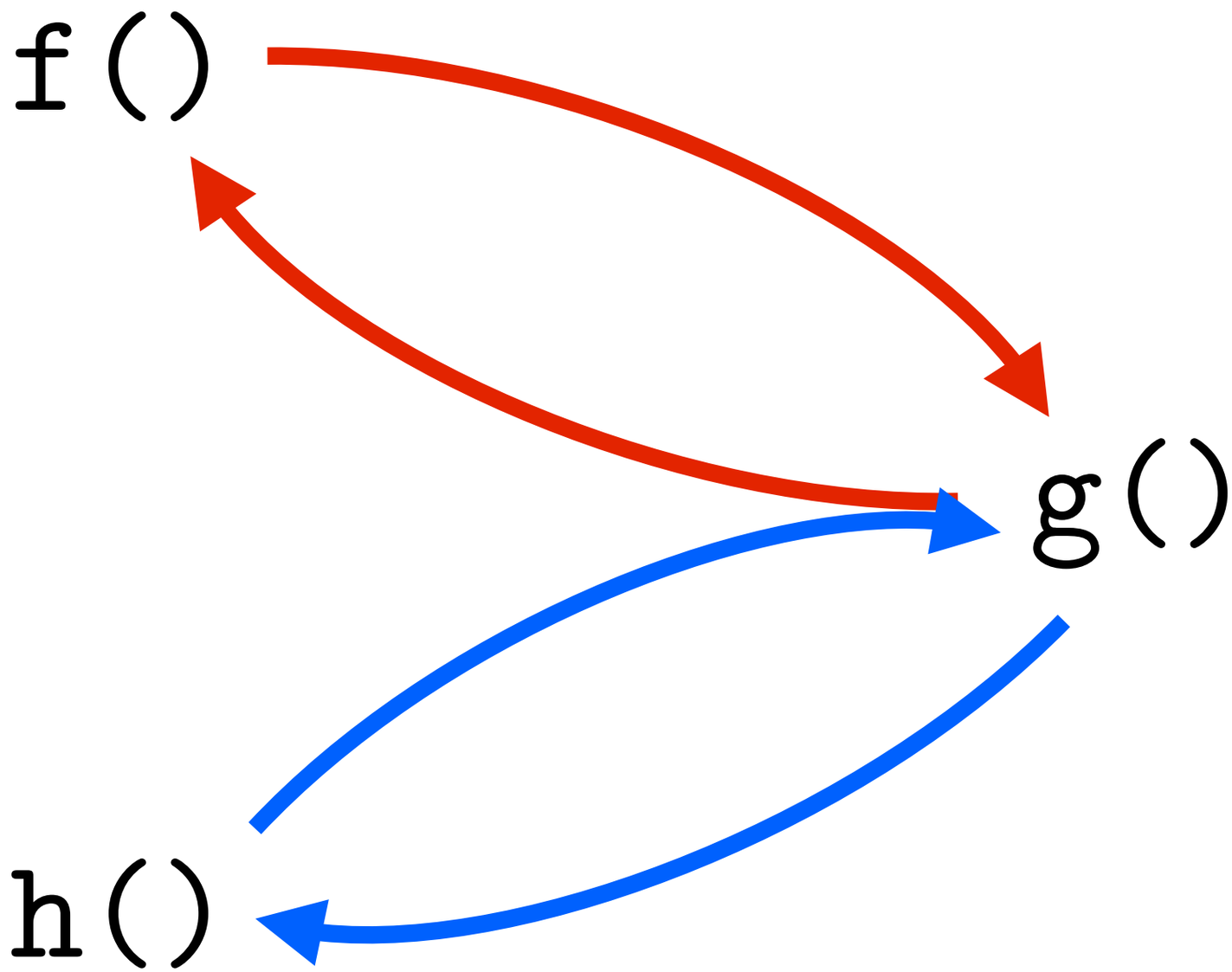
f()
g()
h()

f()

g()

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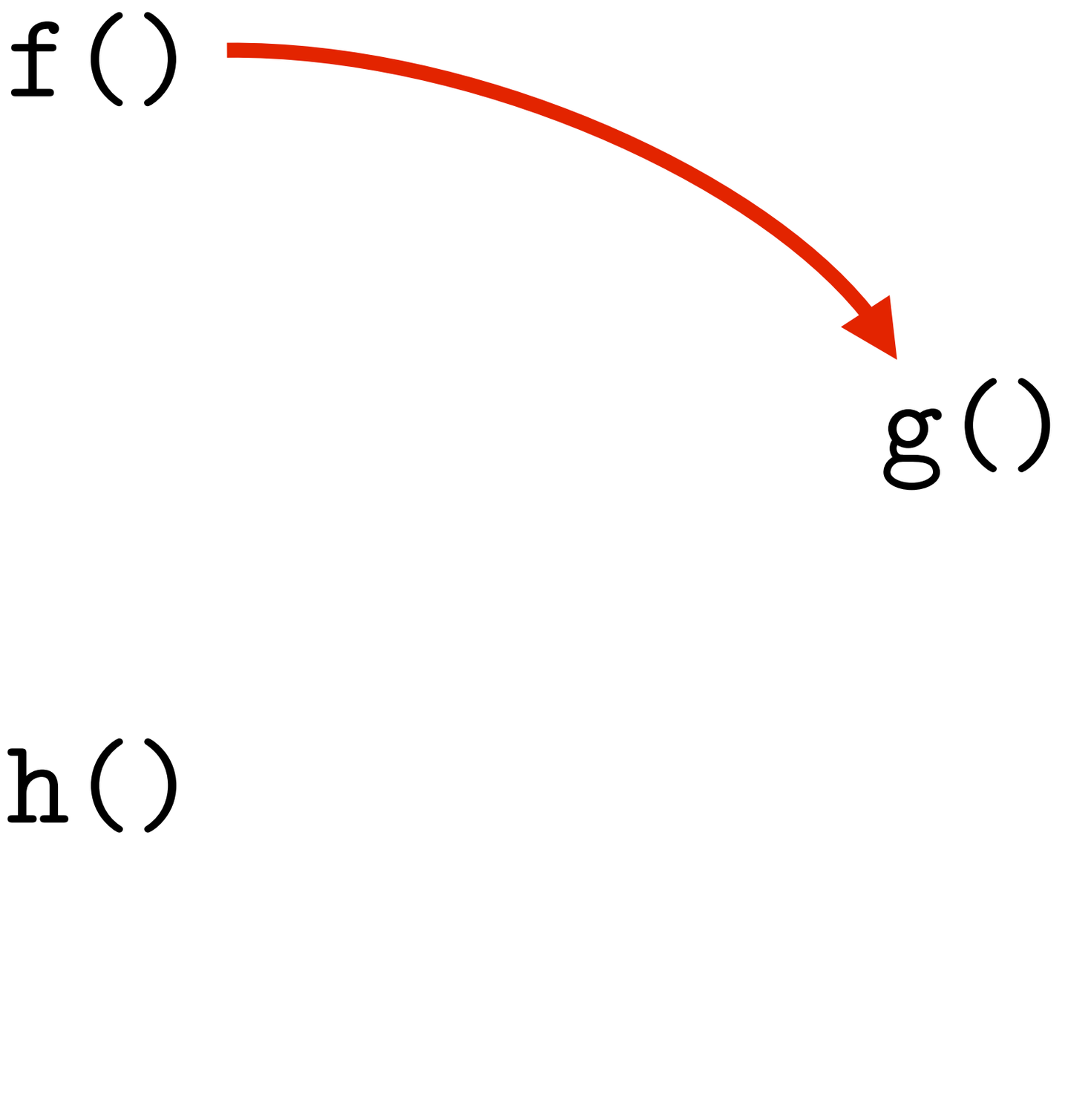
f()
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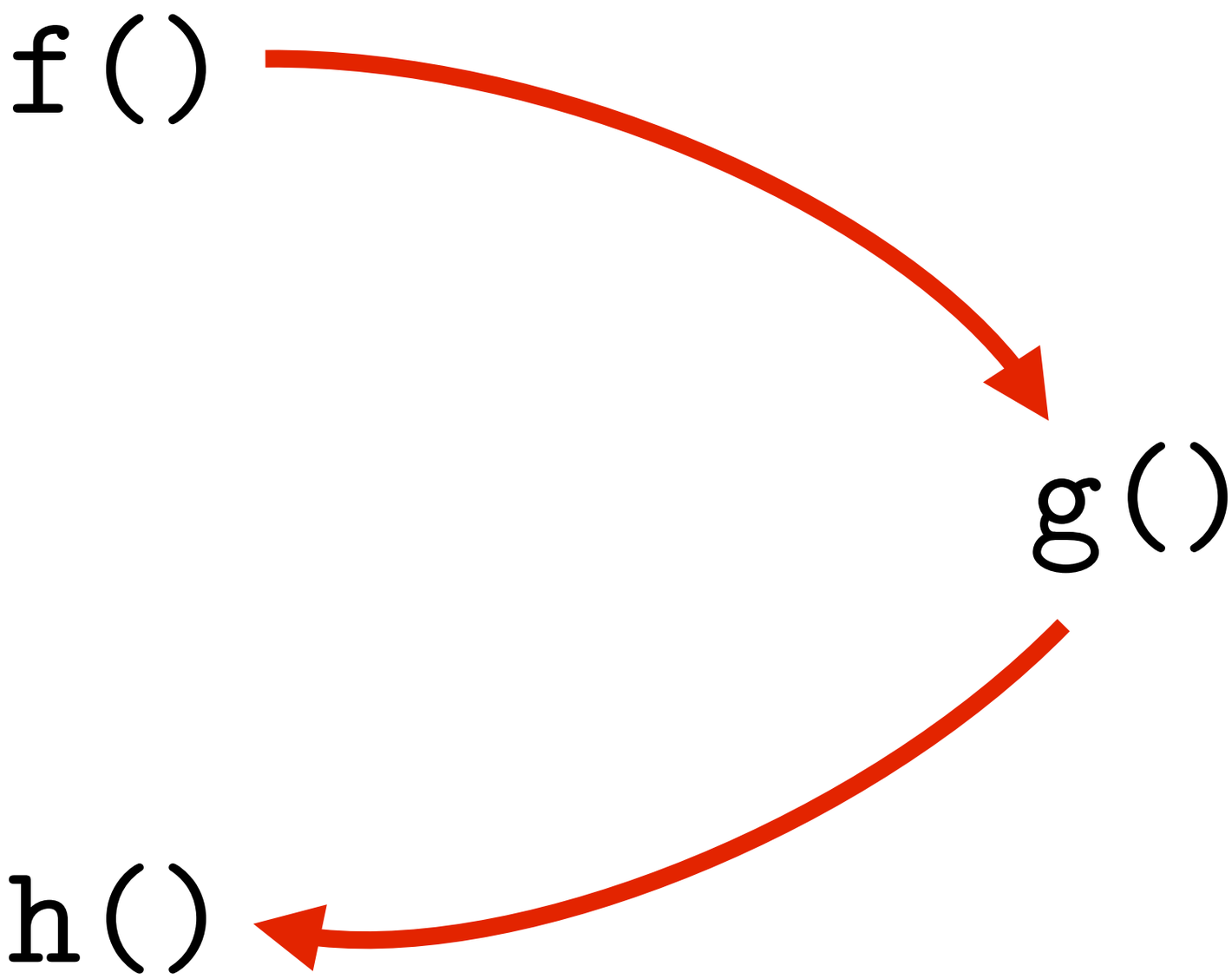
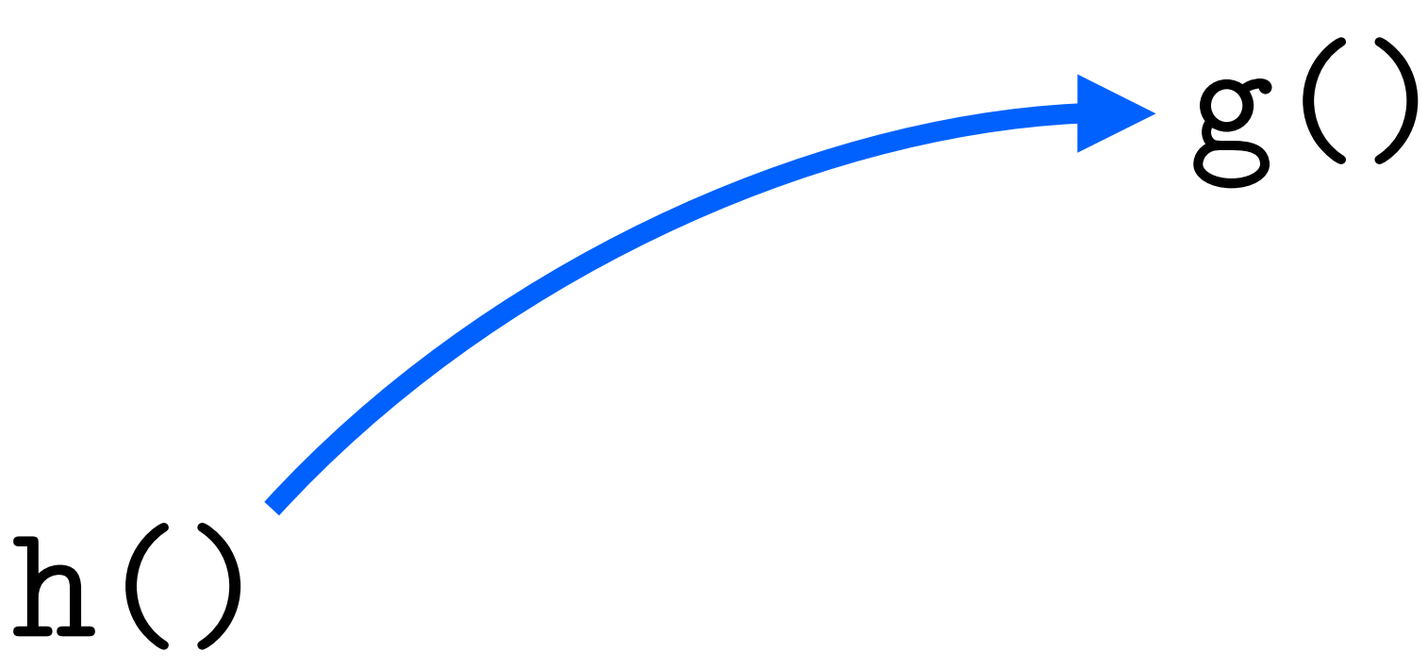

f()
g()
h()

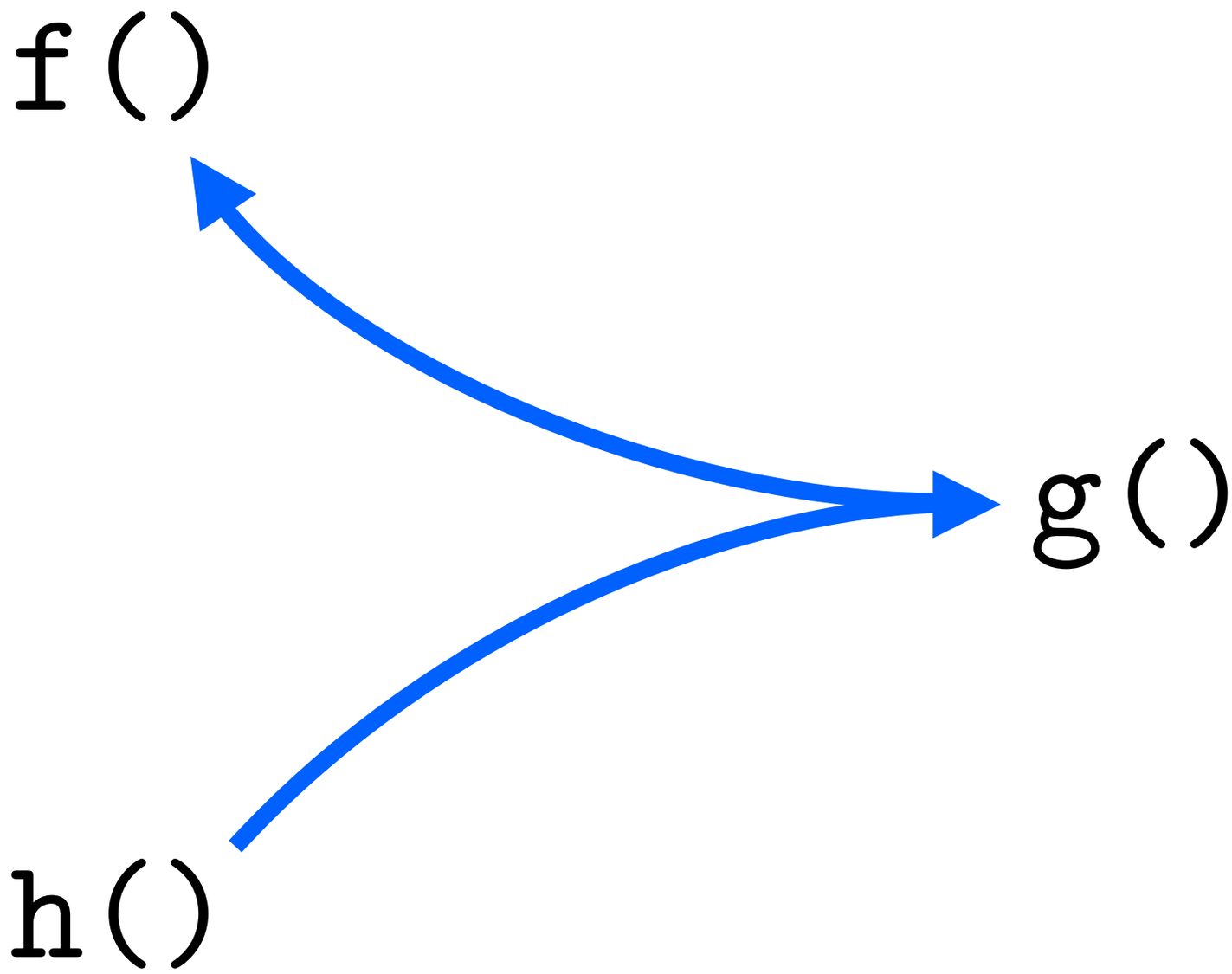
f()

g()

h()

f()




f()
g()
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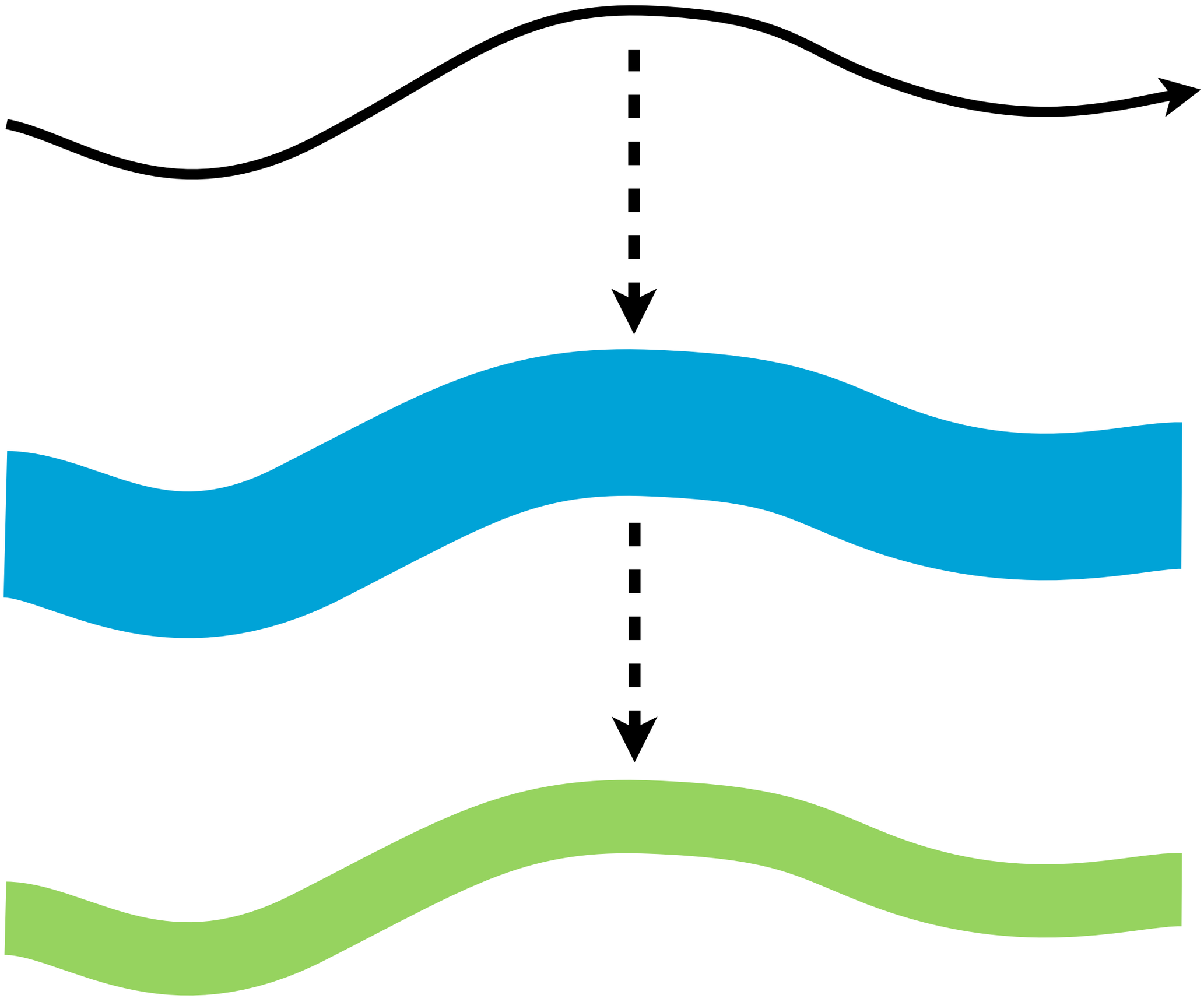
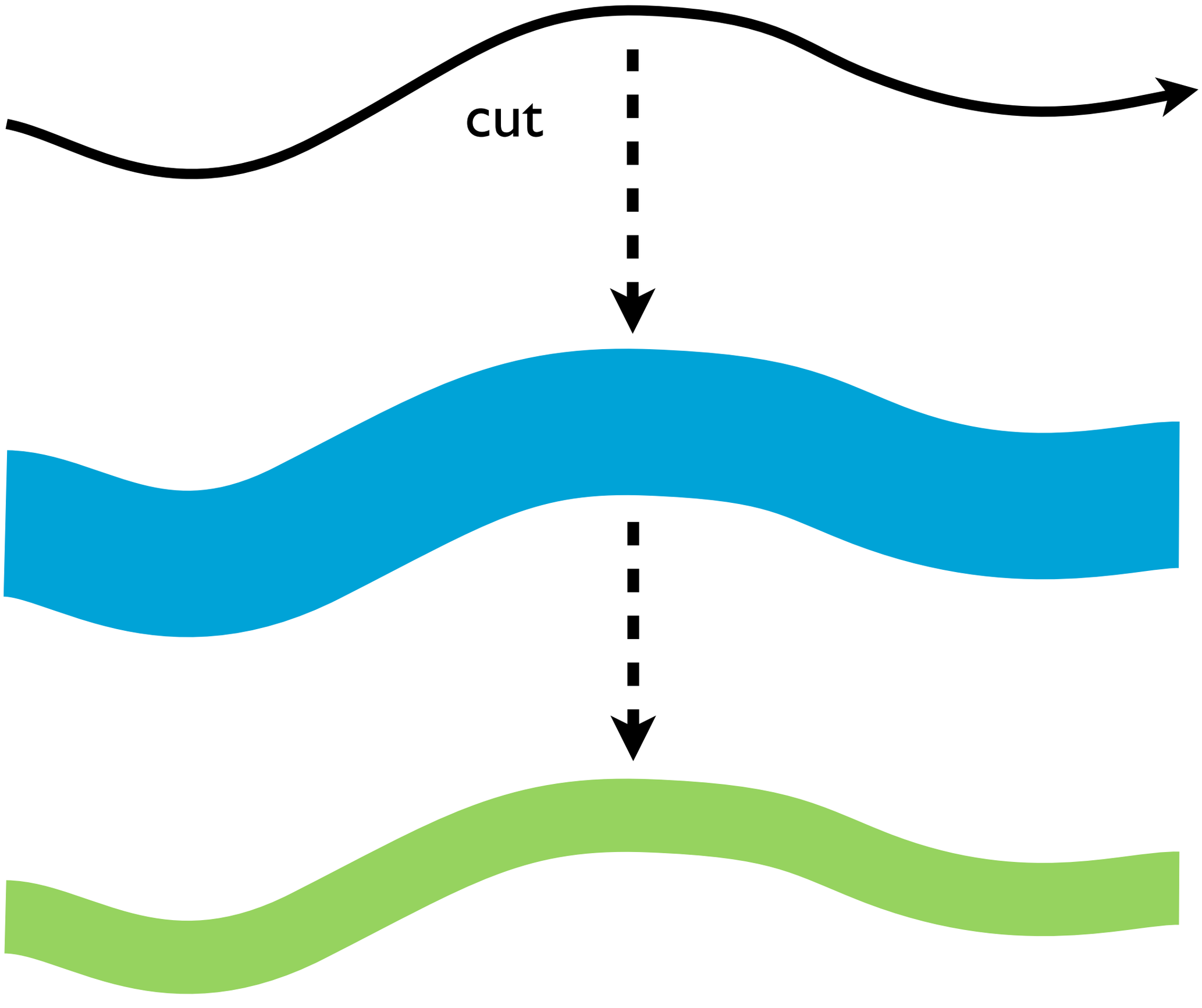
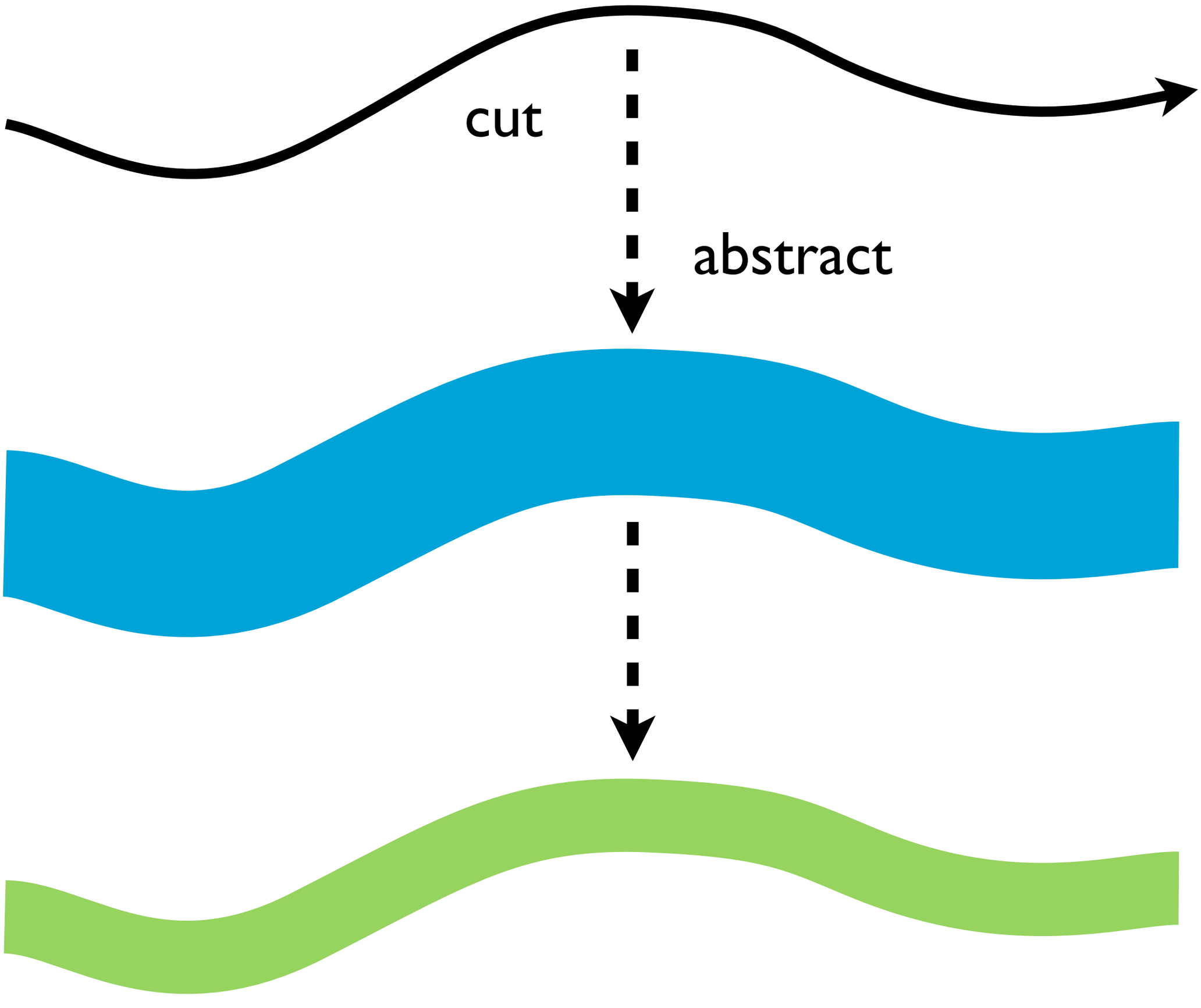
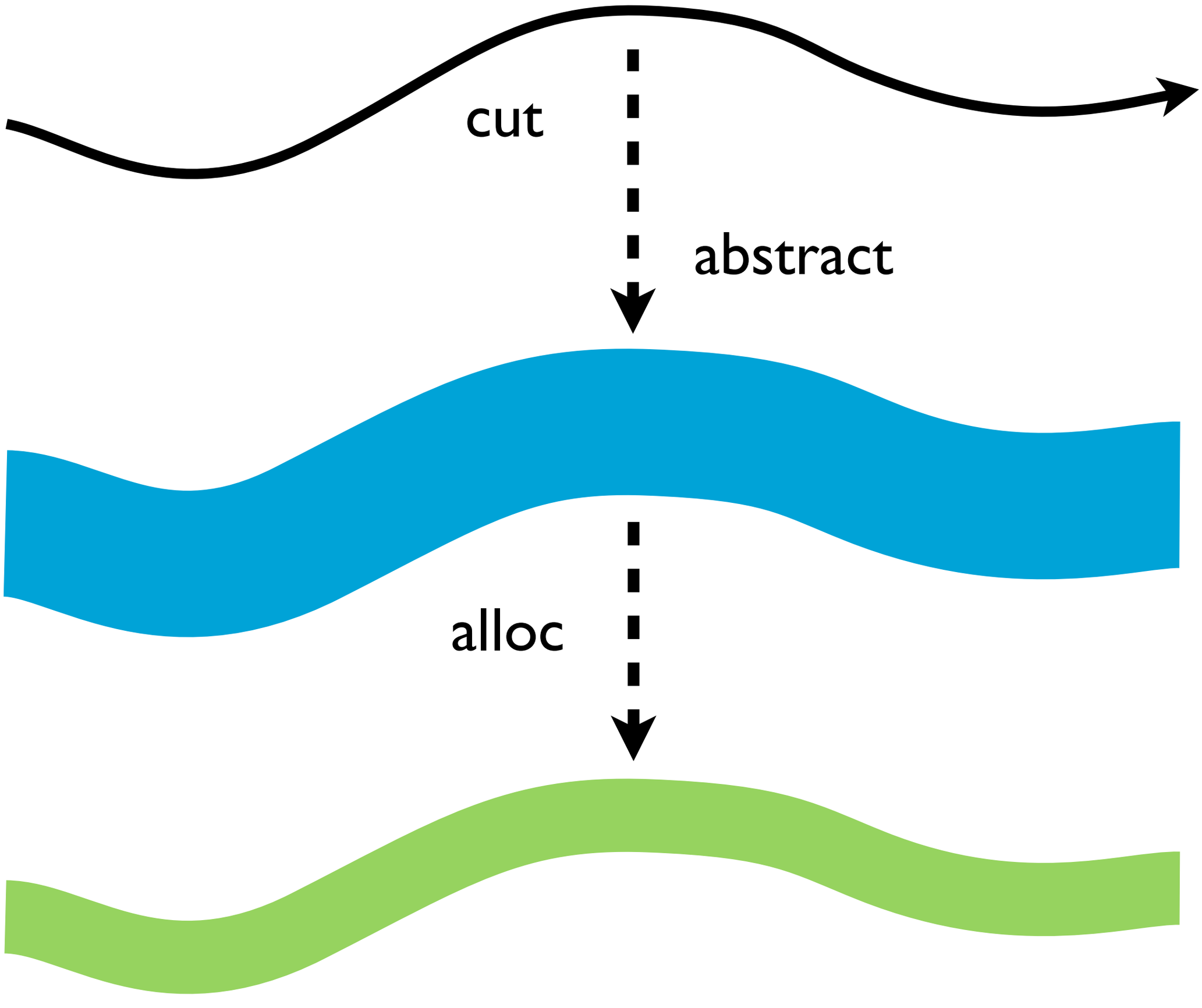



cut


cut
abstract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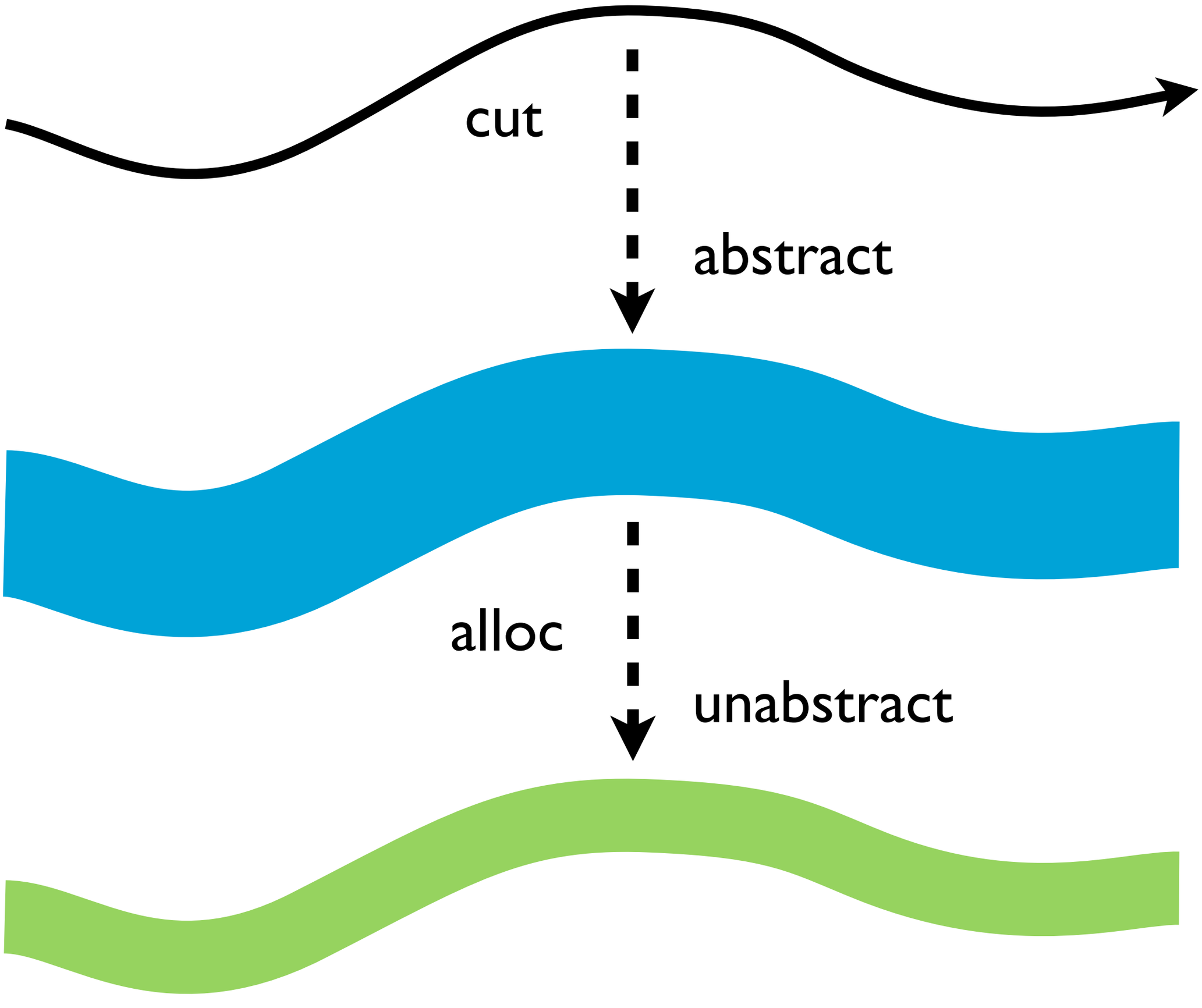
abstract
alloc


cut
abstract
alloc
unabstract

Talk!

Talk!

More in ICFP 2010, SAS 2010, Scheme 2010, SAS 2011