# A Brief History of the Freedom of Expressions

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# Sapir-Whorf Hypothesis

Edward Sapir

*Language limits thought.*

Benjamin Whorf

# Programming languages

- Where did they come from?
- What kinds of languages do we have now?
- What will they do for us tomorrow?

# What is a programming language?

A **programming language** is a language that can describe a transformation from inputs to outputs...

# What is a programming language?

A **programming language** is a language that can describe a transformation from inputs to outputs...

...without ambiguity.

# Origins of programming languages

# Origins of mathematical notation

# Euclid’s algorithm (300 BC)

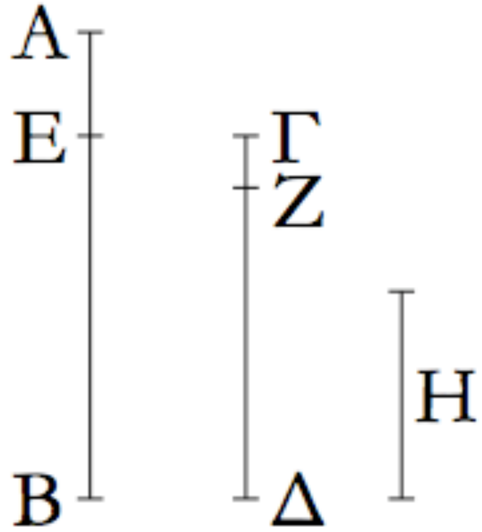
- Greatest common divisor
- GCD(24,18) = 6
- GCD(4,6) = 2

(Probably) Euclid

# 300 BC

β΄.

Δύο ἀριθμῶν δοθέντων μὴ πρώτων πρὸς ἀλλήλους τὸ μέγιστον αὐτῶν κοινὸν μέτρον εὑρεῖν.




Ἔστωσαν οἱ δοθέντες δύο ἀριθμοὶ μὴ πρῶτοι πρὸς ἀλλήλους οἱ ΑΒ, ΓΔ. δεῖ δὴ τῶν ΑΒ, ΓΔ τὸ μέγιστον κοινὸν μέτρον εὑρεῖν.

Εἰ μὲν οὖν ὁ ΓΔ τὸν ΑΒ μετρεῖ, μετρεῖ δὲ καὶ ἑαυτόν, ὁ ΓΔ ἄρα τῶν ΓΔ, ΑΒ κοινὸν μέτρον ἐστίν. καὶ φανερόν, ὅτι καὶ μέγιστον· οὐδεὶς γὰρ μείζων τοῦ ΓΔ τὸν ΓΔ μετρήσει.

Εἰ δὲ οὐ μετρεῖ ὁ ΓΔ τὸν ΑΒ, τῶν ΑΒ, ΓΔ ἀνθυφαιρουμένου ἀεὶ τοῦ ἐλάσσονος ἀπὸ τοῦ μείζονος λειφθήσεταί τις ἀριθμός, ὃς μετρήσει τὸν πρὸ ἑαυτοῦ. μονὰς μὲν γὰρ οὐ λειφθήσεται· εἰ δὲ μή, ἔσονται οἱ ΑΒ, ΓΔ πρῶτοι πρὸς ἀλλήλους· ὅπερ οὐχ ὑπόκειται. λειφθήσεταί τις ἄρα ἀριθμὸς, ὃς μετρήσει τὸν πρὸ ἑαυτοῦ. καὶ ὁ μὲν ΓΔ τὸν ΒΕ μετρῶν λειπέτω ἑαυτοῦ ἐλάσσονα τὸν ΕΑ, ὁ δὲ ΕΑ τὸν ΔΖ μετρῶν λειπέτω ἑαυτοῦ ἐλάσσονα τὸν ΖΓ, ὁ δὲ ΓΖ τὸν ΑΕ μετρείτω. ἐπεὶ οὖν ὁ ΓΖ τὸν ΑΕ μετρεῖ, ὁ δὲ ΑΕ τὸν ΔΖ μετρεῖ, καὶ ὁ ΓΖ ἄρα τὸν ΔΖ μετρήσει. μετρεῖ δὲ καὶ ἑαυτόν· καὶ ὅλον ἄρα τὸν ΓΔ μετρήσει. ὁ δὲ ΓΔ τὸν ΒΕ μετρεῖ· καὶ ὁ ΓΖ ἄρα τὸν ΒΕ μετρεῖ· μετρεῖ δὲ καὶ τὸν ΕΑ· καὶ ὅλον ἄρα τὸν ΒΑ μετρήσει· μετρεῖ δὲ καὶ τὸν ΓΔ· ὁ ΓΖ ἄρα τοὺς ΑΒ, ΓΔ μετρεῖ. ὁ ΓΖ ἄρα τῶν ΑΒ, ΓΔ κοινὸν μέτρον ἐστίν. λέγω δή, ὅτι καὶ μέγιστον. εἰ γὰρ μή ἐστιν ὁ ΓΖ τῶν ΑΒ, ΓΔ μέγιστον κοινὸν μέτρον, μετρήσει τις τοὺς ΑΒ, ΓΔ ἀριθμοὺς ἀριθμὸς μείζων ὢν τοῦ ΓΖ. μετρείτω, καὶ ἔστω ὁ Η. καὶ ἐπεὶ ὁ Η τὸν ΓΔ μετρεῖ, ὁ δὲ ΓΔ τὸν ΒΕ μετρεῖ, καὶ ὁ Η ἄρα τὸν ΒΕ μετρεῖ· μετρεῖ δὲ καὶ ὅλον τὸν ΒΑ· καὶ λοιπὸν ἄρα τὸν ΑΕ μετρήσει. ὁ δὲ ΑΕ τὸν ΔΖ μετρεῖ· καὶ ὁ Η ἄρα τὸν ΔΖ μετρήσει· μετρεῖ δὲ καὶ ὅλον τὸν ΔΓ· καὶ λοιπὸν ἄρα τὸν ΓΖ μετρήσει ὁ μείζων τὸν ἐλάσσονα· ὅπερ ἐστὶν ἀδύνατον· οὐκ ἄρα τοὺς ΑΒ, ΓΔ ἀριθμοὺς ἀριθμός τις μετρήσει μείζων ὢν τοῦ ΓΖ· ὁ ΓΖ ἄρα τῶν ΑΒ, ΓΔ μέγιστόν ἐστι κοινὸν μέτρον [ὅπερ ἔδει δεῖξαι].

Πόρισμα.

Ἐκ δὴ τούτου φανερόν, ὅτι ἐὰν ἀριθμὸς δύο ἀριθμοὺς μετρῇ, καὶ τὸ μέγιστον αὐτῶν κοινὸν μέτρον μετρήσει· ὅπερ ἔδει δεῖξαι.

# 2008 AD

```
gcd(a, b) = (b == 0) ? a : gcd(b, a mod b)
```

# Limits on the Greeks

- No notation for zero.
- No variables for unknowns.
- No symbols for operations.
- Long division required Ph.D.
- Irrational numbers punished by death.

# Example

- The number such that four of its roots is equal to its three of its square.
- $4x = 3x^2$.

# Indian numerals (596)

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 |
|---|---|---|---|---|---|---|---|---|---|
| १ | २ | ३ | ४ | ५ | ६ | ७ | ८ | ९ | ० |
| Nagari numerals around 11th century A.D. | | | | | | | | | |

- Notation for zero.
- Decimal numerals.
- Calculation easier.

Brahmagupta

# Solving quadratics (820)



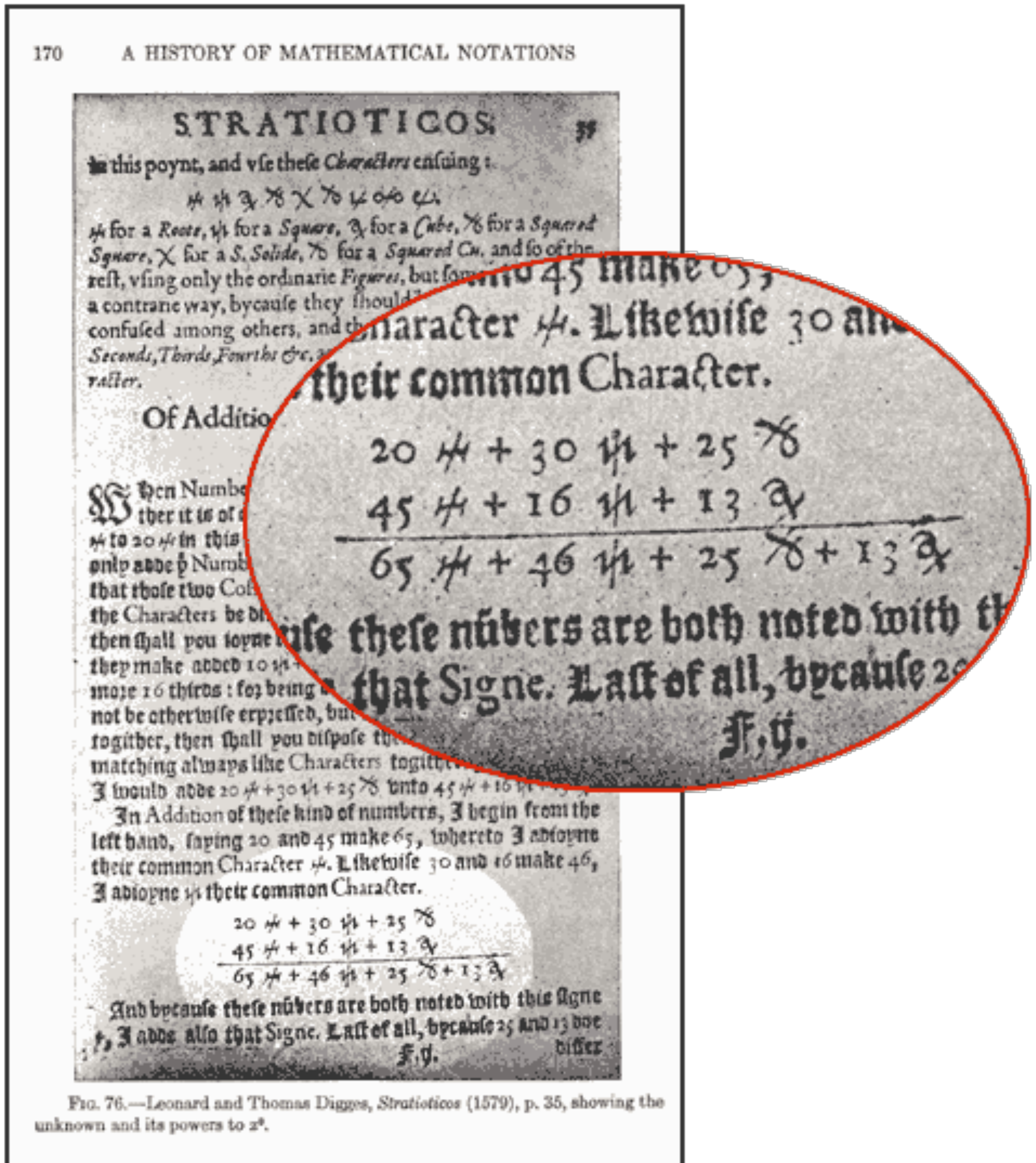
Muhammad ibn Mūsā al-Khwārizmī

# Solving quadratics (820)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$


Muhammad ibn Mūsā al-Khwārizmī

# Variables (1570s)





François Viète

# Example

- $3 + x$ is equal to 10 times $x^2$.
- $3 + x = 10x^2$

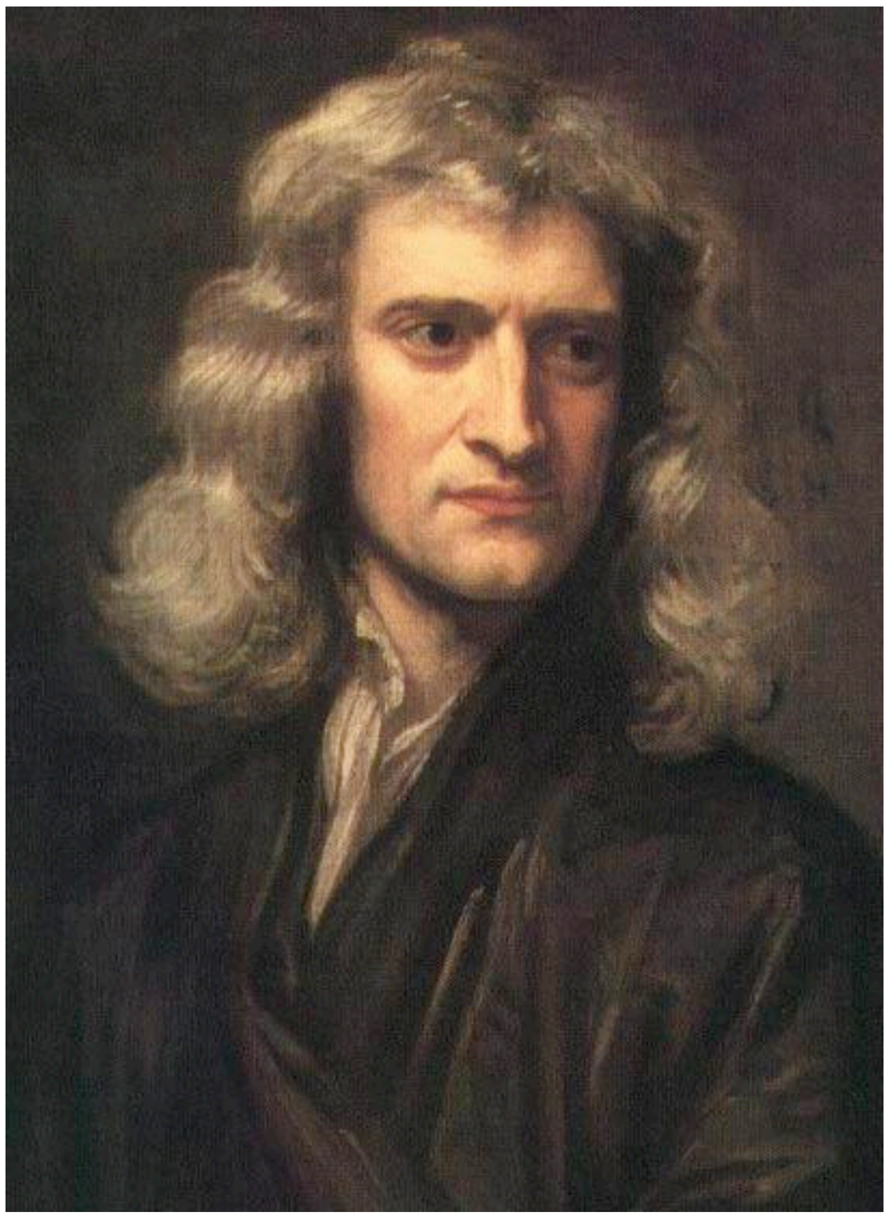
# Operations (Early 1600s)

- Letters for variables.
- Symbols for operations.
- Led to slide rule.

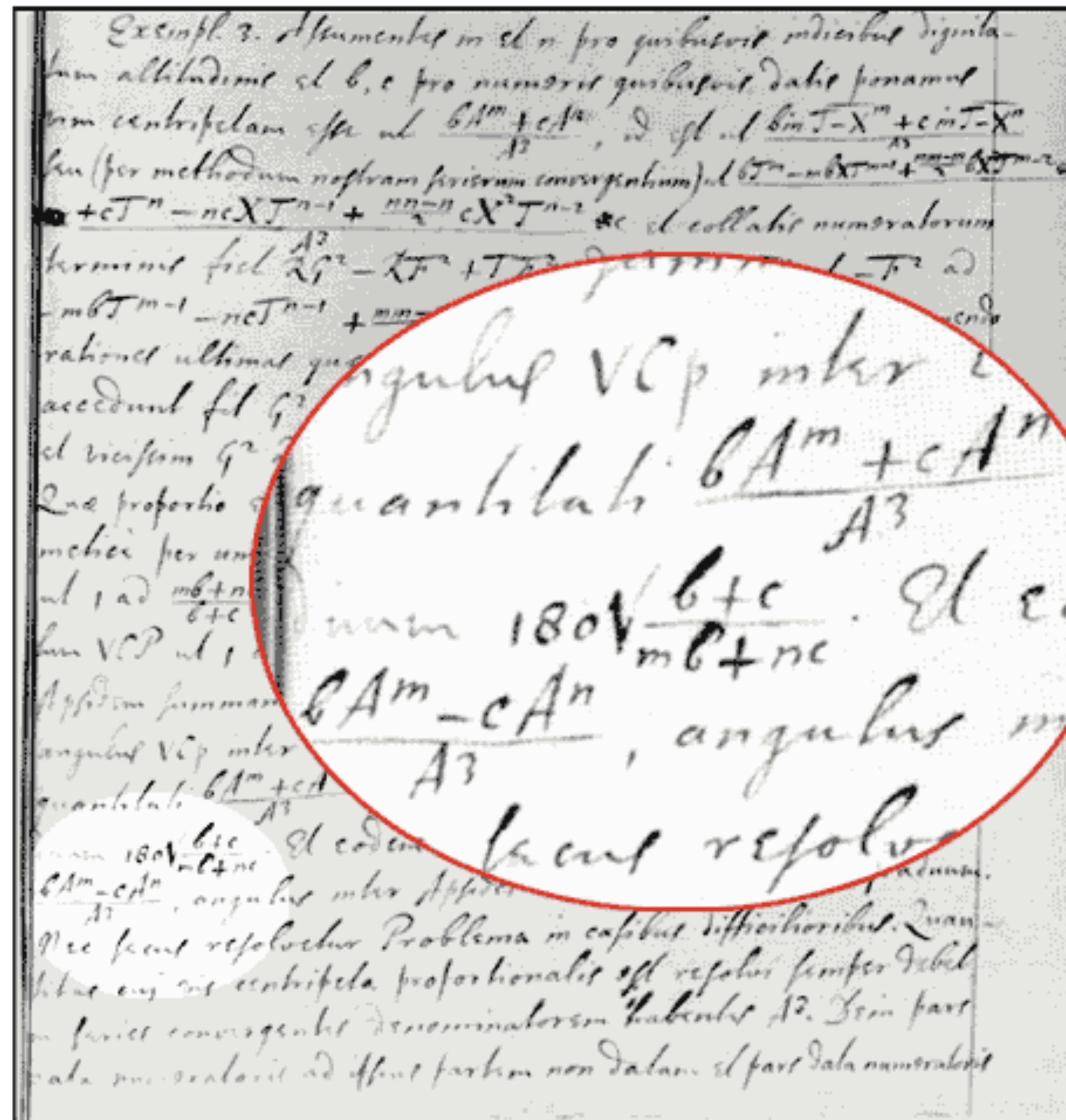
| Symbols | Meanings of Symbols |
|---|---|
| | *Commens. potentia* |
| | *Incommens. potentia* |
| | *Rationale* |
| | *Irrationale* |
| | Medium |
| | Line, cut extr. and mean ratio |
| | *Major ejus portio* |
| | *Minor ejus portio* |
| sim | Simile |
| | *Proxime majus* |
| | *Proxime minus* |
| | *Aequale vel minus* |
| | *Aequale vel majus* |
| ▯ | *Rectangulum* |
| □ | *Quadratum* |
| △ | *Triangulum* |
| | *Latus, radix* |
| | *Media proportion* |
| ∽ | *Differentia*[24] |
| ‖ | Parallel |
| log | Logarithm |
| log : Q : | Log. of square |
| s | Sine[25] |
| t | Tangent |
| se | Secant |
| sv | *Sinus versus* |
| s ver | *Sinus versus*[26] |
| sin : com | Sine complement |
| s co | Cosine |
| t co | Cotangent |
| se co | Cosecant |
| sin | Sine |
| tan | Tangent |
| sec | Secant |
| sec : parall | Sum of secants |

William Oughtred

# Calculus (Late 1600s)



Isaac Newton



*Principia* (1687)

# Calculus (Late 1600s)

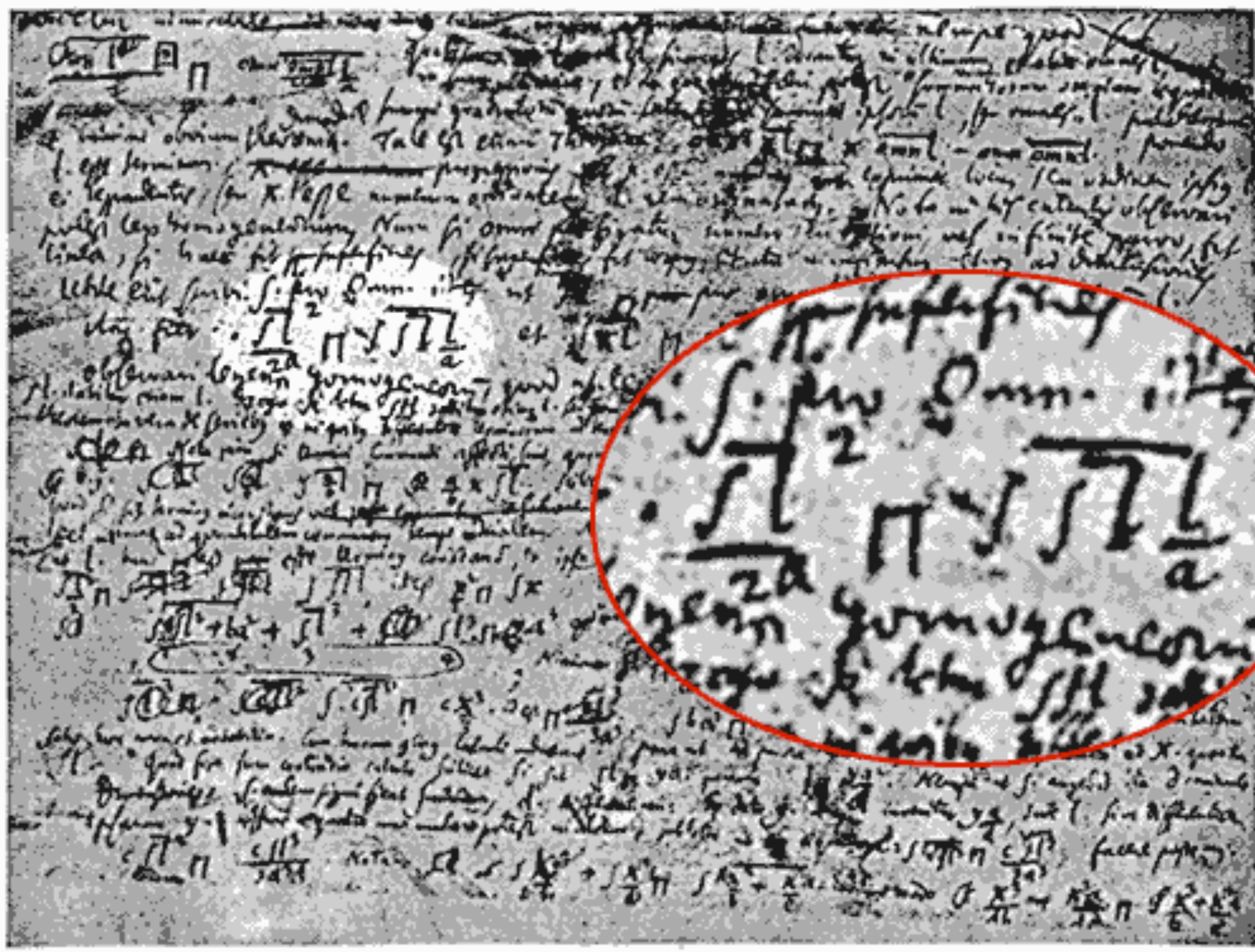
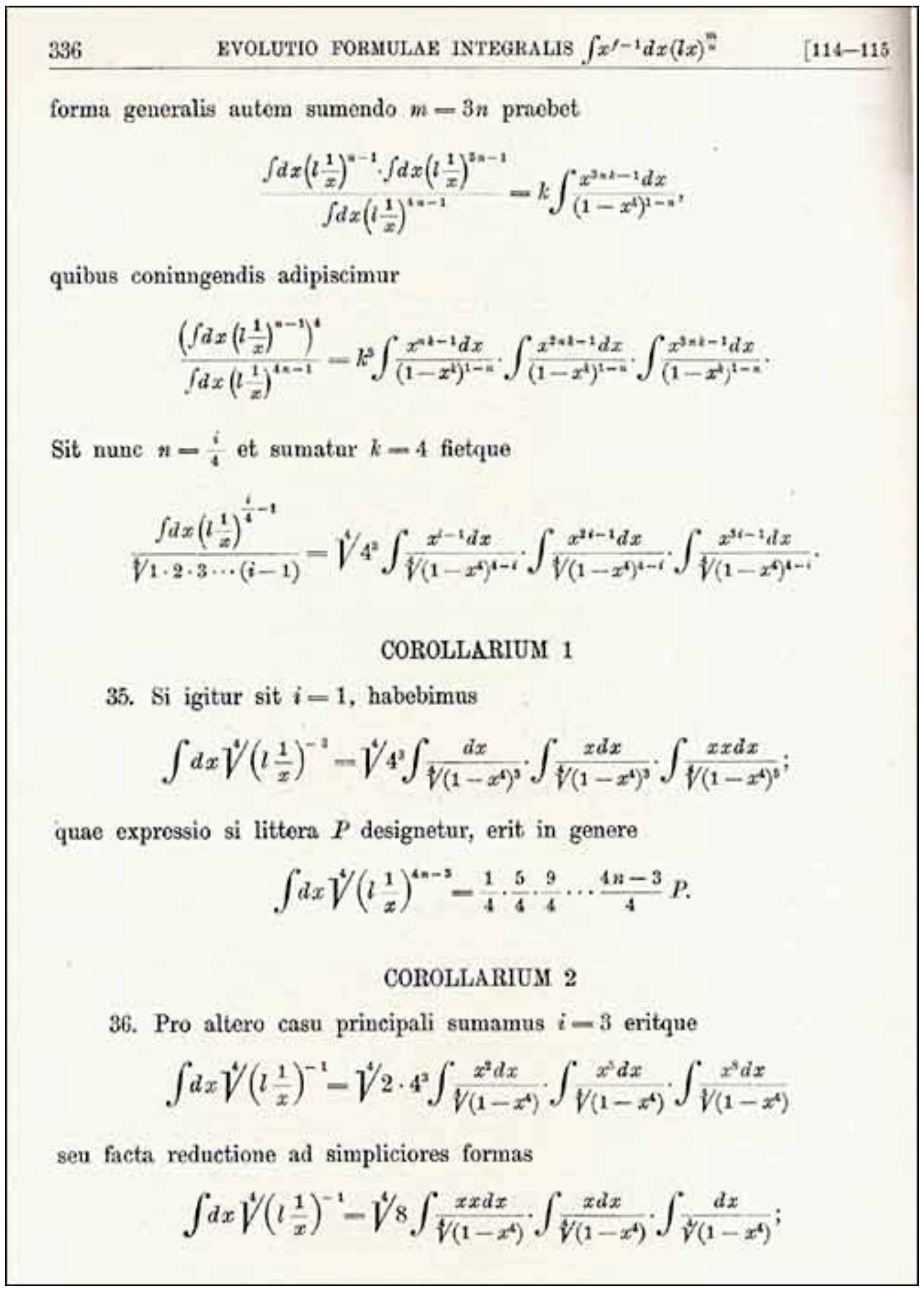


FIG. 124.—Facsimile of manuscript of Leibniz, dated Oct. 29, 1675, in which his sign of integration first appears. (Taken from C. I. Gerhardt's *Briefwechsel von G. W. Leibniz mit Mathematikern* [1899].)

Gottfried Leibniz

# Euler (1700s)





forma generalis autem sumendo $m = 3n$ praebet

$$\frac{\int dx\left(l\frac{1}{x}\right)^{n-1}\cdot\int dx\left(l\frac{1}{x}\right)^{3n-1}}{\int dx\left(l\frac{1}{x}\right)^{4n-1}} = k\int\frac{x^{3nk-1}dx}{(1-x^k)^{1-n}},$$

quibus coniungendis adipiscimur

$$\frac{\left(\int dx\left(l\frac{1}{x}\right)^{n-1}\right)^4}{\int dx\left(l\frac{1}{x}\right)^{4n-1}} = k^3\int\frac{x^{nk-1}dx}{(1-x^k)^{1-n}}\cdot\int\frac{x^{2nk-1}dx}{(1-x^k)^{1-n}}\cdot\int\frac{x^{3nk-1}dx}{(1-x^k)^{1-n}}.$$

Sit nunc $n = \frac{i}{4}$ et sumatur $k = 4$ fietque

$$\frac{\int dx\left(l\frac{1}{x}\right)^{\frac{i}{4}-1}}{\sqrt[4]{1\cdot 2\cdot 3\cdots(i-1)}} = \sqrt[4]{4^3}\int\frac{x^{i-1}dx}{\sqrt[4]{(1-x^4)^{4-i}}}\cdot\int\frac{x^{2i-1}dx}{\sqrt[4]{(1-x^4)^{4-i}}}\cdot\int\frac{x^{3i-1}dx}{\sqrt[4]{(1-x^4)^{4-i}}}.$$

COROLLARIUM 1

35. Si igitur sit $i = 1$, habebimus

$$\int dx\sqrt[4]{\left(l\frac{1}{x}\right)^{-3}} = \sqrt[4]{4^3}\int\frac{dx}{\sqrt[4]{(1-x^4)^3}}\cdot\int\frac{xdx}{\sqrt[4]{(1-x^4)^3}}\cdot\int\frac{xxdx}{\sqrt[4]{(1-x^4)^3}};$$

quae expressio si littera $P$ designetur, erit in genere

$$\int dx\sqrt[4]{\left(l\frac{1}{x}\right)^{4n-3}} = \frac{1}{4}\cdot\frac{5}{4}\cdot\frac{9}{4}\cdots\frac{4n-3}{4}P.$$

COROLLARIUM 2

36. Pro altero casu principali sumamus $i = 3$ eritque

$$\int dx\sqrt[4]{\left(l\frac{1}{x}\right)^{-1}} = \sqrt[4]{2\cdot 4^3}\int\frac{x^2dx}{\sqrt[4]{(1-x^4)}}\cdot\int\frac{x^5dx}{\sqrt[4]{(1-x^4)}}\cdot\int\frac{x^8dx}{\sqrt[4]{(1-x^4)}}$$

seu facta reductione ad simpliciores formas

$$\int dx\sqrt[4]{\left(l\frac{1}{x}\right)^{-1}} = \sqrt[4]{8}\int\frac{xxdx}{\sqrt[4]{(1-x^4)}}\cdot\int\frac{xdx}{\sqrt[4]{(1-x^4)}}\cdot\int\frac{dx}{\sqrt[4]{(1-x^4)}};$$

# The end of the reign of numbers

# Functions

A **function** transforms an input into an output.

$$f(x) = x^2 + 3$$

| input: $x$ | output: $f(x)$ |
|---|---|
| 0 | 3 |
| 1 | 4 |
| 2 | 7 |

# Functions

A **programming language** is a language that can describe a transformation from inputs to outputs.

# Functions

A **programming language** is a language that can describe functions.

# Names for functions

$$f(x)$$

# Names for functions

$$f$$

# Names for functions

$$f^{(3)}$$

# Sets and Logic (1800s)

- Sets became objects.
- Logic became math.
- Math began unifying.

Giuseppe Peano

# Sets

A **set** is a collection of objects.

- $\{1,2,3\}$ is a set of three numbers.
- $\{\}$ is the empty set.
- $\{\{1,2,3\}\}$ is a set containing a set.
- $\mathbb{N}$ is the set of natural numbers.

# Frege's unification

- Logic as foundation.
- Sets as atoms.
- Numbers from sets.
- Functions from sets.

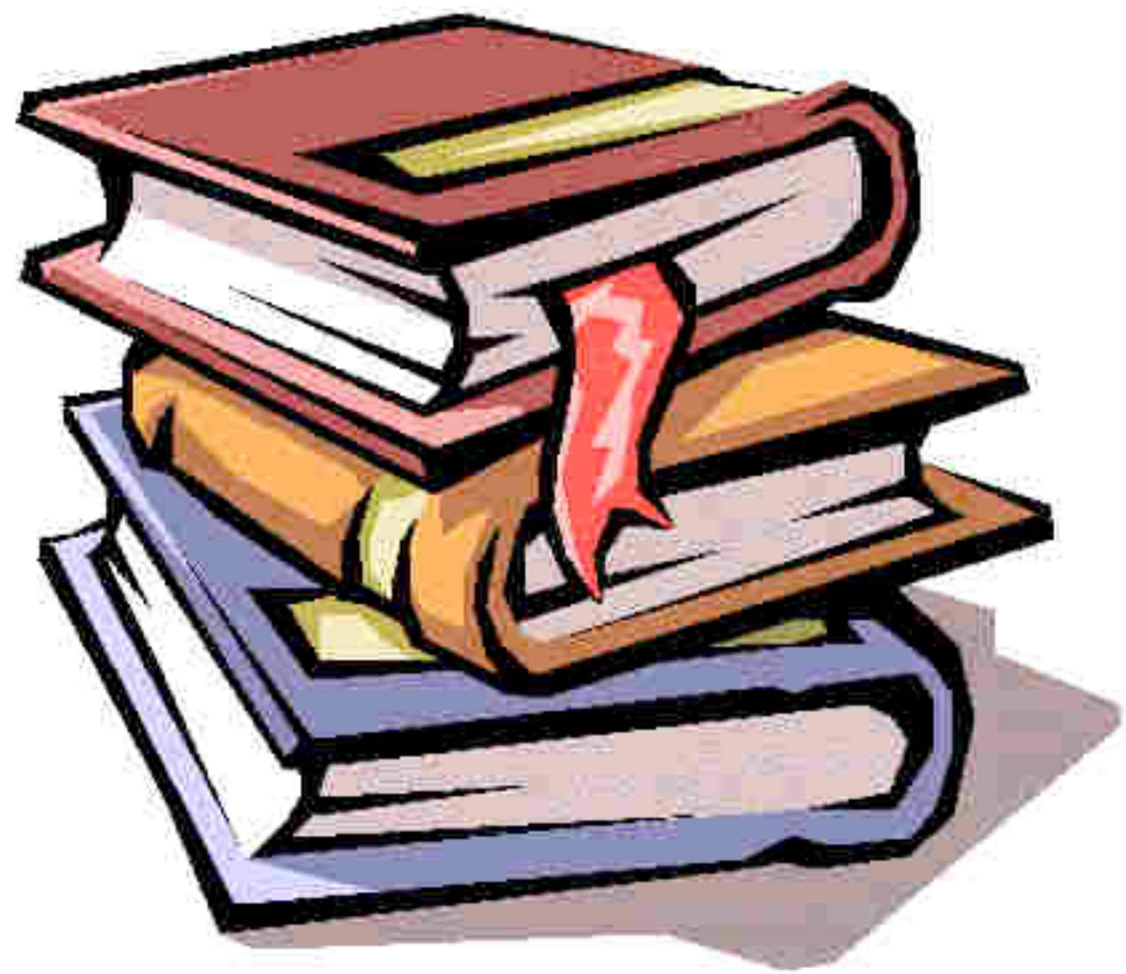
Gottlob Frege

# Example

- Every even integer greater than two can be written as the sum of two primes.
- $\forall n > 2 : \exists a, b : p(a) \wedge p(b) \wedge a+b = n.$

# *Grundgesetze der Arithmetik*

- Published in 1903.
- Foundation for math.



# Russell’s postcard


Dear Frege,

$\{x \mid x \notin x\} \in \{x \mid x \notin x\}$

XOXO,

Bertrand R.

Prof. Dr. Gottlob Frege
University of Göttingen
Göttingen, Germany

# Russell’s postcard


Dear Frege,

U FAIL.

Prof. Dr. Gottlob Frege
University of Göttingen
Göttingen, Germany

XOXO,

Bertrand R.

# Russell's paradox

$$\{X \mid X \notin X\} \in \{X \mid X \notin X\}$$

# Russell's paradox

$$\{X \mid X \notin X\} \in \{X \mid X \notin X\}$$

Does the set of all sets that do not contain themselves contain itself?

# Russell's paradox

$$\{X \mid X \notin X\} \in \{X \mid X \notin X\}$$

Does the set of all sets that do not contain themselves contain itself?

The barber shaves all those that do not shave themselves.

# Russell's paradox

$$\{X \mid X \notin X\} \in \{X \mid X \notin X\}$$

Does the set of all sets that do not contain themselves contain itself?

The barber shaves all those that do not shave themselves.

But, then who shaves the barber?

# Russell's solution: Orders

- Problem is self-reference.
- Example: This sentence is false.
- Solution: Order sentences.
- Must reference lower orders.
- *Seems* to avoid paradox.

Bertrand Russell

# Functions as foundation?

- Notation for functions.
- $f(x) = x^2$
- $f(2) = 4$
- $f = \lambda x.x^2$
- $(\lambda x.x^2)(2) = 4$

Alonzo Church

# Lambda Calculus (1920s)

Throw away everything in math, except:

| | |
|---|---|
| $x$ | variables |
| $f(e)$ | function application |
| $\lambda x.\, e$ | function definition |

# Lambda Calculus

$$(\lambda x.\, e)\ v = \{v/x\}e.$$

# The Lambda Calculus

Can encode:

- Numbers.
- True and false.
- Propositions.
- Sets.
- Recursive functions.
- Logic.
- ...

# Another paradox

$$k = (\lambda x.\neg(x\ x))\ (\lambda x.\neg(x\ x)) = \neg k$$

# Turing machine (1936)

- Student of Church.
- Defined computability.
- Showed λ = computer.

Alan Turing

# Modern programming languages

# Lisp (1958)

- λ as programming language.
- S-Expression notation.
- Code as data.
- Data as code.
- Self-evaluating.

John McCarthy

# Example

```
(define (factorial x)
   (if (= x 0)
       1
       (* n (factorial (- n 1)))))
```

# Kinds of languages

- Declarative: Describes relationship.
- Imperative: Describes process.

# Example: Declarative

- A PBJ is the result of placing peanut butter and jelly between two slices of bread.

# Example: Imperative

- Place slice of bread on table.
- Add peanut butter.
- Add jelly.
- Place slice of bread on top.

# Declarative languages

- Roots in the lambda calculus.
- Examples: Haskell, SML, Lisp/Scheme

- Easy to reason about correctness.
- Generally safe and bug free.
- Can be inefficient.
- Need to think mathematically to use them.

# Imperative languages

- Trace roots to Babbage's engine (1800s).
- Examples: C, C++, Java, Python.

- Hard to know if correct.
- Generally very efficient.
- Extremely dangerous.

<exploding-rocket-video />

PowerPoint

A fatal exception 0E has occurred at 0137:BFFA21C9. The current application will be terminated.

* Press any key to terminate the current application.
* Press CTRL+ALT+DEL again to restart your computer. You will lose any unsaved information in all applicatons.

Press any key to continue _

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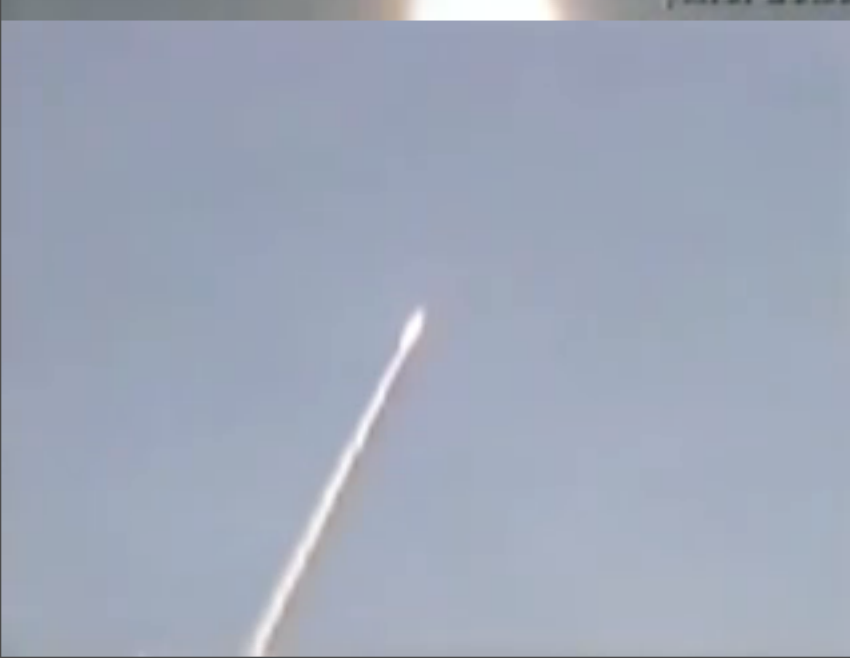
ARIANESPACE



CNN
Interactive



# The future of programming languages

# Front lines of research

- Get the best of both worlds.
- Write program in both; prove equivalent.
- Use a mixed language: Scala, Ynot.

# The future

- Programs won’t crash.
- Programs won’t hang.
- Programs will always do the right thing.

# Thank you

http://matt.might.net/

# Example: Numbers

- $0 = \lambda s.\lambda z.z$
- $n{+}1 = \lambda s.\lambda z.s(n\ s\ z)$

# Example: True, false, if

- $true = \lambda t.\lambda f.t$
- $false = \lambda t.\lambda f.f$
- $if = \lambda b.\lambda t.\lambda f.(b\ t\ f\ )$

# Omega

$$\omega = (\lambda x.x\ x)\ (\lambda x.x\ x) = \omega$$

# Fixed-point Combinator

$$\mathrm{Fix}(F) = F(\mathrm{Fix}(F))$$

# Fixed points and self reference

- $x^2 = 1 + x$
- $x = x^2$ - $1$
- $x = \mathrm{F}(x)$, where $\mathrm{F} = \lambda x.x^2$ - $1$
- $x = \mathrm{Fix}(\mathrm{F}) = \pm 1$

# Recursion

- $f(n) = \text{if } n{=}0 \text{ then } 1 \text{ else } n \times f(n\text{-}1)$
- $F(f) = \lambda n. n{=}0 \text{ then } 1 \text{ else } n \times f(n\text{-}1)$
- $! = \mathrm{Fix}(F)$

# Fixed-point Combinator

$$\text{Fix} = \lambda f.(\lambda x.f\ (x\ x))\ (\lambda x.f\ (x\ x))$$